5th Grade Math

ELG 5.MD.C Understand concepts of volume and relate volume to multiplication and to addition

Vertical Progression:

3rd Grade	**3.MD.C Geometric measurement: understanding concepts of area and relate area to multiplication and to addition.** ○ **3.MD.C.5** Recognize area as an attribute of plane figures and understand concepts of area measurement. ○ **3.MD.C.5.a** A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. ○ **3.MD.C.5.b** A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. ○ **3.MD.C.7** Relate area to the operations of multiplication and addition. ○ **3.MD.C.7.a** Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. ○ **3.MD.C.7.b** Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. ○ **3.MD.C.7.c** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning. ○ **3.MD.C.7.d** Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
4th Grade	**4.MD.A Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** ○ **4.MD.A.3** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.
5th Grade	**5.MD.C Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** ○ **5.MD.C.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. ○ **5.MD.C.3.a** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. ○ **5.MD.C.3.b** A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. ○ **5.MD.C.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. ○ **5.MD.C.5** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. ○ **5.MD.C.5.a** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. ○ **5.MD.C.5.b** Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

	○ **5.MD.C.5.c** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.
6^{th} Grade	**6.G.A Solve real world and mathematical problems involving area, surface area, and volume.** ○ **6.G.A.2** Find the volume of a right rectangular prism by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find the volumes of right rectangular prisms with fractional edge lengths in the context of solving real world and mathematical problems.

Students will demonstrate command of the ELG by:

- Measuring volumes by counting cubes, using cubic centimeters, cubic inches, cubic feet, and improvised units.
- Finding volumes of right rectangular prism with whole number side lengths by packing them with unit cubes and showing the volume is the same as would be found by multiplying the edge lengths or equivalently by multiplying the height by the area of the base.
- Applying the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in context of solving real world and mathematical problems.

Vocabulary:

- cubic unit
- cubic centimeter, cubic inch, cubic meter
- formula
- right rectangular prism
- unit cube
- volume
- width

Sample Instructional/Assessment Tasks:

1) **Standard: 5.MD.C.5.a**

Source: Illustrative Mathematics
https://www.illustrativemathematics.org/content-standards/5/MD/C/5/tasks/1655

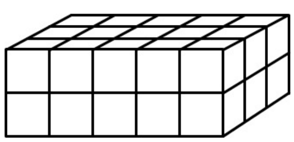
Item Prompt: Using Volume to Understand the Associative Property of Multiplication
Make sure you have plenty of snap cubes.

a. Build a rectangular prism that is 2 cubes on one side, 3 cubes on another, and 5 cubes on the third side.
b. We will say that the volume of one cube is 1 cubic unit. What is the volume of the rectangular prism?
c. Jenna said, **"The rectangular prism is 2 cubes by 3 cubes by 5 cubes, so the volume of the prism is 2 x 3 x 5 cubic units."**
Ari said, **"I don't know what 2 x 3 x 5 means. Do you multiply the 2 and 3 first**
2 x 3 x 5 = (2 x 3) x 5
= 6 x 5
So you have 6 groups of 5 , or do you multiply the 3 and the 5 first
2 x 3 x 5 = 2 x (3 x 5)
= 2 x 15
So you have 2 groups of 15?"
   - Explain how you can see the rectangular prism as being made of 2 groups with 15 cubes in each.
   - Explain how you can also see the rectangular prism as being made of 6 groups with 5 cubes in each.

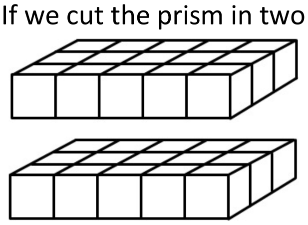
d. Does it matter which numbers you multiple first when you want to find the volume of a rectangular prism?

Solution:

a. A rectangular prism that is 2 cubes on one side, 3 cubes on another, and 5 on the third side might look like this:

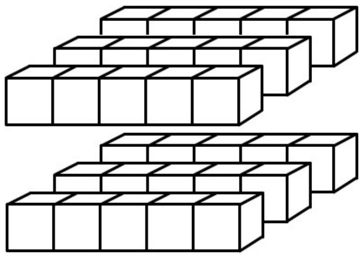


b. The volume of the rectangular prism is 30 cubic units.
c. If we cut the prism in two layers:



we can see it as being made of 2 groups with 15 cubes in each.

If we cut the prism into two layers and then cut each layer into three slices:



we can see the prism as being made up of 6 slices with 5 cubes in each.

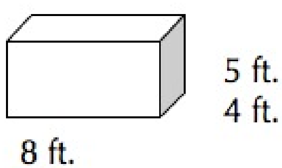
d. It doesn't matter which numbers you multiply first when you want to find the volume of a rectangular prism. We can see this most easily when the side-lengths are whole numbers, because we can always cut the prism into layers (and then each layer into slices) to show the same phenomenon that we saw here. We can write this in symbols as follows:
$V = l \times w \times h$
$= l \times (w \times h)$
$=(l \times w) \times h$

2) Standard(s): 5.MD.C.5.b

Source: Illustrative Mathematics
https://www.illustrativemathematics.org/content-standards/5/MD/C/5/tasks/1308

Item Prompt: Cari's Aquarium
Cari is the lead architect for the city's new aquarium. All of the tanks in the aquarium will be rectangular prisms where the side lengths are whole numbers.

a. Cari's first tank is 4 feet wide, 8 feet long and 5 feet high. How many cubic feet of water can her tank hold?



b. Cari knows that a certain species of fish needs at least 240 cubic feet of water in their tank. Create 3 separate tanks that hold exactly 240 cubic feet of water. (Ex: She could design a tank that is 10 feet wide, 4 feet long and 6 feet in height.)
c. In the back of the aquarium, Cari realizes that the ceiling is only 10 feet high. She needs to create a tank that can hold exactly 100 cubic feet of water. Name one way that she could build a tank that is not taller than 10 feet.

Correct Answer(s)

a. The tank can hold 160 cubic feet of water because:
Length x Width x Height = Volume
8 feet x 4 feet x 5 feet – 160 feet^3
Alternatively, the base of the tank is:
4 feet x 8 feet = 32 feet^2
Therefore,
Area of Base x Height – Volume
32 feet^2 x 5 feet = 160 feet^3

b. There are many possible solutions to this task. The whole number factor combinations are listed below. Please note that each set of factors is listed only once, though a student could reassign which factor represents length, width or height as long as she or he uses the same three factors.

Length	Width	Height	Volume
240 ft.	1 ft.	1 ft.	240 ft^3
120 ft.	2 ft.	1 ft.	240 ft^3
80 ft.	3 ft.	1 ft.	240 ft^3
60 ft.	4 ft.	1 ft.	240 ft^3
60 ft.	2 ft.	2 ft.	240 ft^3
48 ft.	5 ft.	1 ft.	240 ft^3
40 ft.	6 ft.	1 ft.	240 ft^3
40 ft.	3 ft.	2 ft.	240 ft^3
30 ft.	4 ft.	1 ft.	240 ft^3
30 ft.	2 ft.	2 ft.	240 ft^3
24 ft.	10 ft.	1 ft.	240 ft^3
24 ft.	5 ft.	2 ft.	240 ft^3
20 ft.	12 ft.	1 ft.	240 ft^3
20 ft.	6 ft.	2 ft.	240 ft^3
20 ft.	4 ft.	3 ft.	240 ft^3
16 ft.	15 ft.	1 ft.	240 ft^3
16 ft.	5 ft.	3 ft.	240 ft^3
15 ft.	4 ft.	4 ft.	240 ft^3
15 ft.	8 ft.	2 ft.	240 ft^3
12 ft.	5 ft.	4 ft.	240 ft^3
12 ft.	10 ft.	2 ft.	240 ft^3
10 ft.	6 ft.	4 ft.	240 ft^3
10 ft.	8 ft.	3 ft.	240 ft^3
8 ft.	6 ft.	5 ft.	240 ft^3

Again the teacher should prompt students to think about what their tank would look like in real-life. A tank that is 40 feet long, 3 feet wide and 2 feet high would be very long, skinny and short. This tank would not be good for bigger fish, but it could be useful to watch something small that does not need a lot of space to move, such as a turtle.

c. There are many possible solutions to this task. All whole number factor combinations are listed below to ensure that students understand that they cannot simply choose any three factors that multiply to 100 $ft.^3$ without considering the restrictions on height. Make sure that students think through the real-world implications of this problem: Cari's tank would work as long as it is not higher than 10 feet tall.

Length	Width	Height	Volume
100 ft.	1 ft.	1 ft.	100 $ft.^3$
1 ft.	100 ft.	1 ft.	100 $ft.^3$
50 ft.	2 ft.	1 ft.	100 $ft.^3$
2 ft.	50 ft.	1 ft.	100 $ft.^3$
50 ft.	1 ft.	2 ft.	100 $ft.^3$
50 ft.	2 ft.	1 ft.	100 $ft.^3$
25 ft.	4 ft.	1 ft.	100 $ft.^3$
25 ft.	1 ft.	4 ft.	100 $ft.^3$
1 ft.	25 ft.	4 ft.	100 $ft.^3$
4 ft.	25 ft.	1 ft.	100 $ft.^3$
25 ft.	2 ft.	2 ft.	100 $ft.^3$
2 ft.	25 ft.	2 ft.	100 $ft.^3$
20 ft.	5 ft.	1 ft.	100 $ft.^3$
5 ft.	20 ft.	1 ft.	100 $ft.^3$
10 ft.	1 ft.	10 ft.	100 $ft.^3$
1 ft.	10 ft.	10 ft.	100 $ft.^3$
10 ft.	10 ft.	1 ft.	100 $ft.^3$
10 ft.	5 ft.	2 ft.	100 $ft.^3$
10 ft.	2 ft.	5 ft.	100 $ft.^3$
5 ft.	2 ft.	10 ft.	100 $ft.^3$
5 ft.	10 ft.	2 ft.	100 $ft.^3$
2 ft.	10 ft.	5 ft.	100 $ft.^3$
2 ft.	5 ft.	10 ft.	100 $ft.^3$
5 ft.	4 ft.	5 ft.	100 $ft.^3$
4 ft.	5 ft.	5 ft.	100 $ft.^3$
5 ft.	5 ft.	4 ft.	100 $ft.^3$

3) **Standard: 5.MD.C.5.c**

Source: Illustrative Mathematics
https://www.illustrativemathematics.org/content-standards/5/MD/C/5/tasks/1971

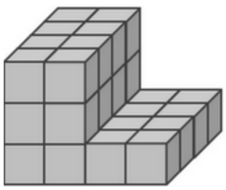
Item Prompt: Breaking Apart Composite Solids

Students will need two different color markers or crayons to complete this task.

John was finding the volume of this figure. He decided to break it apart into two separate rectangular prisms. John found the volume of the solid below using this expression:

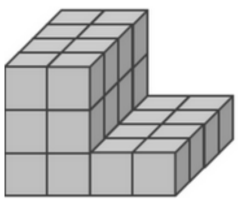
(4 x 4 x 1) + (2 x 4 x 2).

Decompose the figure into two rectangular prisms and shade them in different colors to show one way John might have thought about it.



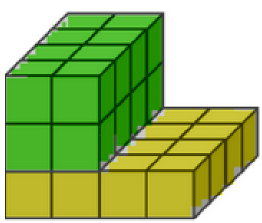
Phillis also broke this solid into two rectangular prisms, but she did it differently than John. She found the volume of the solid below using this expression: (2 x 4 x 3) + (2 x 4 x 1).

Decompose the figure into two rectangular prisms and shade them in different colors to show one way Phillis might have thought about it.



Solution:

John's picture could be:



Phillis' picture could be: