ELG 5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions

Vertical Progression:

3rd Grade	**3.NF.A Develop understanding of fractions as numbers.** ○ **3.NF.A.1** Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. ○ **3.NF.A.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. ○ **3.NF.A.3.d** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols > , =, or < , and justify the conclusions, e.g., by using a visual fraction model.
4th Grade	**4.NF.A Extend understanding of fraction equivalence and ordering.** ○ **4.NF.A.1** Explain why a fraction *a/b* is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **4.NF.B Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.** ○ **4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. ○ **4.NF.B.4.a** Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* ○ **4.NF.B.4.b** Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* ○ **4.NF.B.4.c** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*
5th Grade	**5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** ○ **5.NF.B.3** Interpret a fraction as division of the numerator by the denominator $(a/b = a \div b)$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* ○ **5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. ○ **5.NF.B.4.a** Interpret the product $(a/b) \times q$ as a parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* ○ **5.NF.B.4.b** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by

	multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. ○ **5.NF.B.5** Interpret multiplication as scaling (resizing), by: ○ **5.NF.B.5.a** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. ○ **5.NF.B.5.b** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. ○ **5.NF.B.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. ○ **5.NF.B.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. ○ **5.NF.B.7.a** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* ○ **5.NF.B.7.b** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* ○ **5.NF.B.7.c** Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*
6th Grade	**6.RP.A Understand ratio concepts and use reasoning to solve problems.** ○ **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities**.** ○ **6.RP.A.2** Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. **6.NS.A Apply and extend previous understandings of multiplication and division to divide fractions by fractions.** ○ **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

Students will demonstrate command of the ELG by:

- Interpreting fractions as dividing numerators by denominators.
- Finding areas of rectangles with fraction side lengths by tiling them with unit squares of appropriate unit fraction side lengths.
- Calculating area by multiplying side lengths to find areas of rectangles and representing fraction products as rectangular areas.
- Interpreting multiplication as scaling (resizing).
- Comparing sizes of products to size of one factor on the basis of the size of the other factor, without performing indicated multiplication.

- Explaining why multiplying given numbers by fractions greater than one results in products greater than the given numbers.
- Explaining why multiplying given numbers by fractions less than one results in products smaller than the given numbers.
- Interpreting and computing division of unit fractions by nonzero whole numbers and division of whole numbers by unit fractions.
- Computing quotients of unit fractions by nonzero whole numbers and quotients of whole numbers by unit fractions.
- Solving word problems using visual models or equations involving adding and subtracting fractions, dividing whole numbers leading to answers in the form of fractions or mixed numbers, multiplying fractions and mixed numbers, dividing unit fractions by nonzero whole numbers, and dividing whole numbers by unit fractions.

Vocabulary:

- area
- denominator
- equation
- factor
- fraction
- fractional side lengths
- improper fraction
- mixed number
- multiplication
- numerator
- operations
- product
- proper fraction
- quotient
- remainder
- scaling
- unit fraction
- visual fraction model
- whole number

Sample Instructional/Assessment Tasks:

1) **Standard(s): 5.NF.B.3**

Source: Illustrative Mathematics
https://www.illustrativemathematics.org/content-standards/5/NF/B/3/tasks/858

Item Prompt: How Much Pie?

After a class potluck, Emily has three equally sized apple pies left and she wants to divide them into eight equal portions to give to eight students who want to take some pie home.

a. Draw a picture showing how Emily might divide the pies into eight equal portions. Explain how your picture shows eight equal portions.

b. What fraction of a pie will each of the eight students get?

c. Explain how the answer to (b) is related to the division problem 3 ÷ 8.

Correct answer:

a. shows 3 circles, each divided into 8 equal parts; with each of the eight students getting 1 piece from each pie

b. 3/8

c. 3 divided by 8 is 3/8 which is the portion each student gets

2) Standard(s): 5.NF.B.5

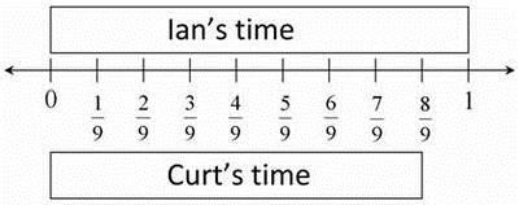
Source: Illustrative Mathematics
https://www.illustrativemathematics.org/content-standards/5/NF/B/5/tasks/22

Item Prompt: Running a Mile

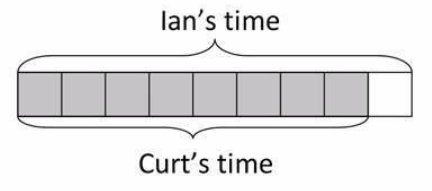
Curt and Ian both ran a mile. Curt's time was $\frac{8}{9}$ Ian's time. Who ran faster? Explain and draw a picture.

Correct Answer:

To find Curt's time, you would multiply Ian's time by $\frac{8}{9}$. Since we are multiplying Ian's time by a number less than 1, Curt's time will be less than Ian's time. The picture shows Ian's time multiplied by 1 above the number line and Ian's time multiplied by $\frac{8}{9}$ below the number line.



Since they both ran the same distance but Curt ran it in less time, he must have been running faster. Curt's time is × Ian's time. That means that if you divide Ian's time into 9 equal time intervals and take 8 of those intervals, you will have Curt's time. So Curt's time to run a mile is less than Ian's and he must be going faster.



3) Standard(s) 5.NF.B.7.b

Source: Illustrative Mathematics
https://www.illustrativemathematics.org/content-standards/5/NF/B/7/tasks/958

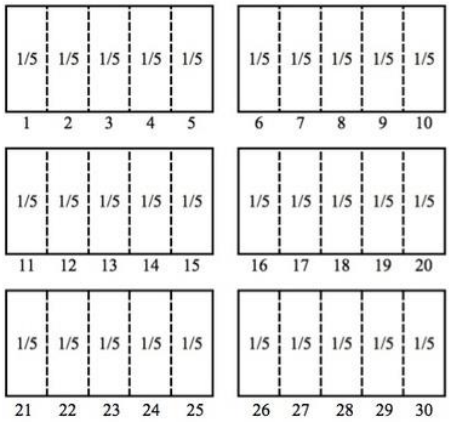
Item Prompt: Origami Stars

Avery and Megan are cutting paper to make origami stars. they need $\frac{1}{5}$ of a sheet of paper in order to make each star. If they have 6 sheets of paper, how many stars can they make? Explain your work and draw a picture to support your reasoning.

Correct Answer: In order to find out how many origami stars Avery and Megan can make, we need to find out how many pieces equivalent to $\frac{1}{5}$ of a sheet of paper we can divide the 6 pieces of paper into. This means that the answer to this question will be the solution to the following division problem:

$6 \div \frac{1}{5} = ?$

Based on our understanding of fractions, we know that there are 5 pieces of that are equivalent to $\frac{1}{5}$ of a sheet in each sheet of paper. This means that there are $6\times5=30$ pieces of paper with size $\frac{1}{5}$ of a sheet in the 6 sheets of paper that Avery and Megan have to make their stars. We know that this is true because $30\times\frac{1}{5}=6$. Based on our understanding of the relationship between multiplication and division, this tells us that $6\div\frac{1}{5}=30$. Thus, Avery and Megan can make 30 origami stars. The picture below shows that this is correct:



The picture above shows that there are 30 pieces of paper with size $\frac{1}{5}$ of a sheet in the 6 sheets of paper that Avery and Megan have to make their stars. This supports our reasoning and shows that Avery and Megan can make 30 origami stars.