ELG 6.2: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Vertical Progression:

4th Grade	**NF.B Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.** o **4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
5th Grade	**5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** o **5.NF.B.3** Interpret a fraction as division of the numerator by the denominator (a/b = a÷b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. o **5.NF.B.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
6th Grade	**ELG 6.2 Apply and extend previous understandings of multiplication and division to divide fractions by fractions.** o **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
7th Grade	**ELG 7.2 Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** o **7.NS.A.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. o **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with rational numbers.

Students will demonstrate command of the ELG by:

- Interpreting a real life problem involving division of fractions and computing using the correct operation.
- Explaining the relationship between their model and equation.
- Justifying why the answer is a reasonable solution to the problem.
- Explaining and using an algorithm to solve fraction division problems.
- Creating a story context that will represent a real life problem that involves division of fractions by fractions.

Vocabulary:

- denominator
- dividend
- divisor
- equation
- improper fraction
- mixed number
- multiplicative inverse
- numerator
- product
- proper fraction
- quotient
- reciprocal

Sample Instructional/Assessment Tasks:

1) **Standard(s): 6.NS.A.1**

Source: North Carolina DPI

Item Prompt:

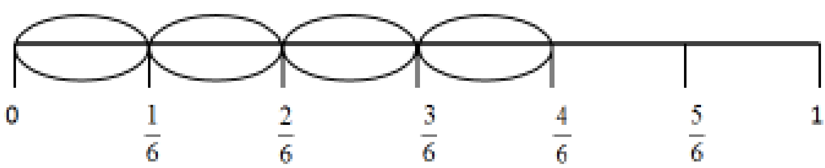
Susan has $\frac{2}{3}$ of an hour left to make cards. It takes her about $\frac{1}{6}$ of an hour to make each card. About how many can she make?

Solution:

This problem can be modeled using a number line.

a. Start with a number line divided into thirds.

b. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.



c. Each circled part represents $\frac{1}{6}$. There are four sixths in two-thirds; therefore, Susan can make 4 cards.

Source NC DPI

2) **Standard(s): 6.NS.A.1**

Source: www.illustrativemathematics.org

Item Prompt:

Dan observes that

$$\frac{6}{10} \div \frac{2}{10} = 6 \div 2$$

He says,

I think that if we are dividing a fraction by a fraction with the same denominator, then we can just divide the numerators.

Is Dan's conjecture true for all fractions? Explain how you know.

Solution:

Solution: A procedural explanation and a conceptual one.

Yes, Dan's rule is correct for all fractions.

Explaining with analogies: *One way of explaining why the rule is correct is to bear in mind that $\frac{6}{10}$ refers to six "items", where the "item" is $\frac{1}{10}$.*

$$\frac{6}{10} = 6\left(\frac{1}{10}\right),$$

so the original division problem can be rephrased as

$$6\left(\frac{1}{10}\right) \div 2\left(\frac{1}{10}\right).$$

The computation

$$6\left(\frac{1}{10}\right) \div 2\left(\frac{1}{10}\right) = 6 \div 2 = 3$$

follows the same logic as:

$$6 \text{ apples} \div 2 \text{ apples} = 6 \div 2 = 3.$$

We would probably not write it as $6\,\text{apples} \div 2\,\text{apples}$; instead we might have a situation where we have 6 apples and want to know how many groups of 2 apples we can make. Since we can make 3 groups with 2 apples in each group, the answer is 3. There was nothing special about the apples, the same reasoning applies to any "objects":

How many groups of 2 peaches can I make if I have 6 peaches?

How many groups of $2 dollars can I make if I have $6?

How many groups of 2 tenths can I make if I have 6 tenths?

When we divide a quantity consisting of m units divided into groups of size n of the same units, then the result does not depend on what the units are. The answer is found by dividing the number m by the number n.

In summary, Dan's rule is true not only for dividing fractions with the same denominator, but also for any division of one quantity (number of units) by another quantity with the same units. In all cases, we find the answer by dividing the numbers, and the kind of unit does not matter.

Explaining with symbols: *Dan might have made his conjecture based on using the "invert and multiply" rule:*

$$\frac{6}{10} \div \frac{2}{10} = \frac{6}{10} \times \frac{10}{2} = \frac{6 \times 10}{2 \times 10} = \frac{6}{2} = 6 \div 2.$$

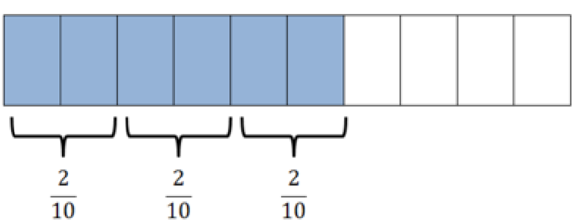
This works just as well for any denominator $d \neq 0$. If m and n are any integers and $n \neq 0$, then:

$$\frac{m}{d} \div \frac{n}{d} = \frac{m}{d} \times \frac{d}{n} = \frac{m \times d}{n \times d} = \frac{m}{n} = m \div n.$$

Submitted by J. Madden. This solution was developed by the 12 middle- and secondary-school teachers in the "LaMSTI On-Ramp Course" at LSU, with major contributions from N. Revaula.

Solution:

A diagram helps make this clear:



There are 6 tenths shaded, and we want to know how many groups of 2 tenths we can make; there are 3 such groups. We can see that there is nothing special about the fact that the small rectangles represent tenths; the same reasoning would work for any denominator.