ELG 8.9: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Vertical Progression:

6th Grade	**ELG 6.8 Solve real-world and mathematical problems involving area, surface area, and volume.** o **6.G.A.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. o **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. o **6.G.A.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
7th Grade	**ELG 7.6 Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** o **7.G.B.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. o **7.G.B.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. o **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
8th Grade	**ELG 8.9 Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.** o **8.G.C.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
Geometry	**ELG.MA.HS.G.13 Explain volume formulas and use them to solve problems.** o **G-GMD.A.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. o **G-GMD.A.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Students will demonstrate command of the ELG by:

- Finding the volumes of cones, cylinders, and spheres.
- Using the volume formulas for cones, cylinders, and spheres to solve real-world problems.

Vocabulary:

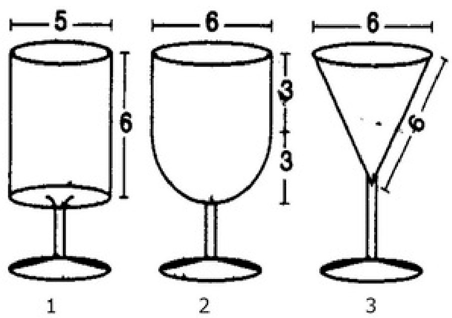
- cone
- cylinder
- diameter
- height
- radius
- sphere
- volume

Sample Instructional/Assessment Tasks:

1) **Standard(s): 8.G.C.9**

Source: https://www.illustrativemathematics.org/content-standards/8/G/C/9/tasks/112

Item Prompt:
The diagram shows three glasses (not drawn to scale). The measurements are all in centimeters.



The bowl of glass 1 is cylindrical. The inside diameter is 5 cm and the inside height is 6 cm.

The bowl of glass 2 is composed of a hemisphere attached to cylinder. The inside diameter of both the hemisphere and the cylinder is 6 cm. The height of the cylinder is 3 cm.

The bowl of glass 3 is an inverted cone. The inside diameter is 6 cm and the inside slant height is 6 cm.

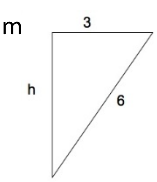
a. Find the vertical height of the bowl of glass 3.

b. Calculate the volume of the bowl of each of these glasses.

c. Glass 2 is filled with water and then half the water is poured out. Find the height of the water.

Correct Answer(s)

The solutions use volume formulas for cylinders, cones and spheres. Note that all units are in terms of centimeters: area units are cm^2, and volume units are cm^3. The units are sometimes omitted for convenience. The diagram cale.

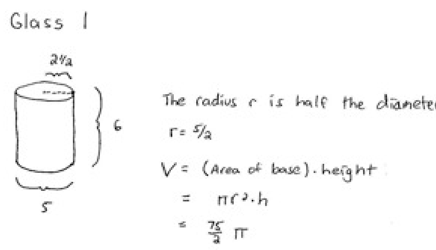


a. To find the vertical height of the bowl portion of the glass, we use the Pythagorean Theorem.

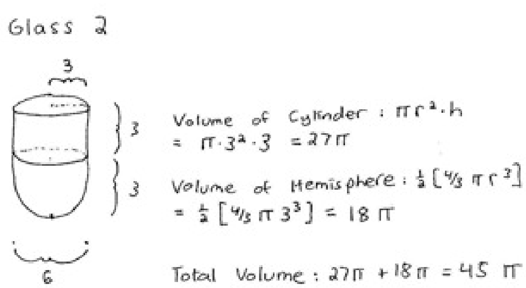
Height in centimeters = $h = \sqrt{6^2 - 3^2} = \sqrt{36 - 9} = \sqrt{27}$. (Some may prefer to write $\sqrt{27} = 3\sqrt{3}$.)

b. For glass 1 we use the fact that the volume of a cylinder is given by V=(area of base)·height. Since the base is a circle with radius r=5^2, the volume V (in cm^3) is

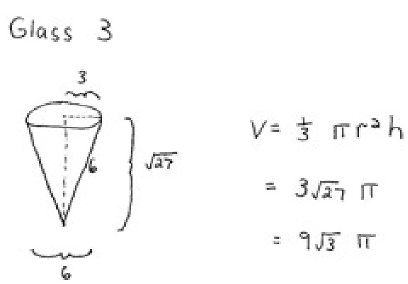
$$V = \left(\pi\left(\frac{5}{2}\right)^2\right)\cdot 6 = \left(\frac{75}{2}\right)\pi = 37\frac{1}{2}\pi.$$



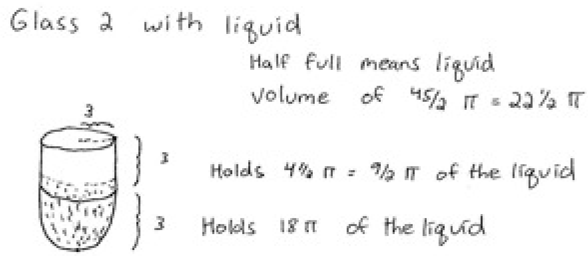
For glass 2 the bowl consists of 2 parts- a cylinder of height and radius 3 which sits atop a hemisphere of radius 3. We add the volumes of these to get the total volume. For the cylinder portion, the volume (in cm^3) is $(\pi 3^2)\cdot 3 = 27\pi$. For the volume of the hemisphere, take half the volume of a sphere of radius 3 to get: $\frac{1}{2}\left(\frac{4}{3}\pi 3^3\right) = \frac{1}{2}(4\cdot 3^2\pi) = 18\pi$. Add these to get the total volume (in cm^3): $27\pi+18\pi=45\pi$.



For the volume of the bowl of glass 3, use the fact that the volume of a cone is given by V = $\frac{1}{3}$ Area of base·Height. The radius is 3 and the height is $\sqrt{27}$ by part (a). So the area of the base is $\frac{1}{3}\pi 3^2$ and the volume V (in cm^3) is $V = \frac{1}{3}\pi 3^2\sqrt{27} = 3\sqrt{27}\pi$. (Or $9\sqrt{3}\pi$.)

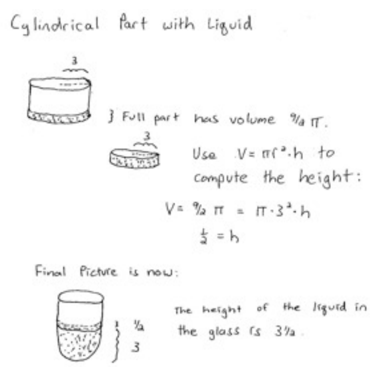


c. Note first that the height of the liquid is measured from the bottom of the bowl of the glass. The total volume of the glass is 45π, so if half the water is poured out then the remaining water occupies a volume of $\frac{45}{2}\pi$, or $22\frac{1}{2}\pi$. Notice that this is not just 3 cm up the glass since the hemispherical part of the glass holds less than the cylindrical part. (It is easy to see that if the glass had a cylindrical bottom, it would be bigger and so have a larger capacity.) As we saw in (b)(ii), the liquid in the glass fills the hemisphere first and then the cylindrical portion of the glass. The hemispherical part of the glass holds a volume of 18π, leaving a volume of $4\frac{1}{2}\pi$, or $\frac{9}{2}\pi$, to fill the cylindrical portion.



The question is now reduced to finding the height of the liquid in the cylindrical portion of the glass. Once we know this height, we can simply add the height from the hemispherical portion, which is 3, and obtain the full height of the liquid in the glass.

To get the height of the liquid in the cylinder we compute the height of a cylinder which has volume $\frac{9}{2}\pi$. We use the formula for volume to solve for the height: $\frac{9}{2}\pi = \pi 3^2 h = 9\pi h$ so $h = \frac{1}{2}$. We can now add 3 to this to get the required height of the liquid in the glass which is (in cm) $3\frac{1}{2}$.

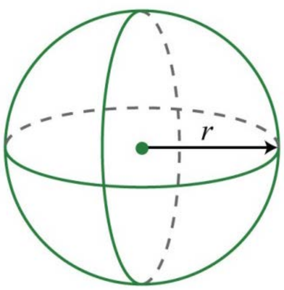
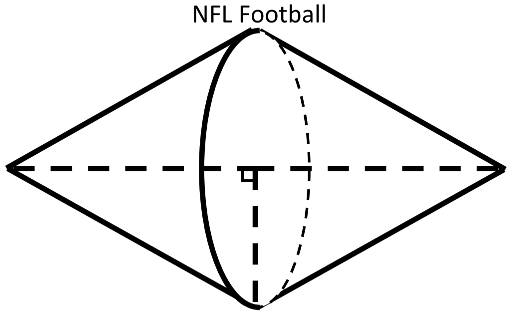


2) **Standard(s): 8.G.C.9**

Source: Jonathan Mattes-Ritz

Item Prompt:
A standard size 5 soccer ball is required to have a circumference close to 27 inches. An official NFL football measures 22 inches around at its widest point and is 11 inches long. Using two cones to approximate the NFL football, determine which ball holds more air and by how much.

Solution:

Size 5 Soccer Ball	NFL Football
 $C = 2\pi r$ $27 = 2\pi r$ $r \approx 4.3\ inches$ $V = \frac{4}{3}\pi r^3$ $V = \frac{4}{3}\pi (8.6)^3$ $V \approx 332\ in^3$	 $C = 2\pi r$ $22 = 2\pi r$ $r \approx 3.5\ inches$ $V = 2\left(\frac{1}{3}\pi r^2 h\right)$ $V = 2\left(\frac{1}{3}\pi (3.5)^2 \left(\frac{11}{2}\right)\right)$ $V \approx 141\ in^3$
$332 - 141 = 191$ The soccer ball is larger by 191 in^3	