Vertical Progression:

7th Grade	**7.EE.B Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** ○ **7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
8th Grade	**8.EE.C Analyze and solve pairs of simultaneous linear equations.** ○ **8.EE.C.8.a** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. ○ **8.EE.C.8.b** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* ○ **8.EE.C.8.c** Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*
Algebra 1	**ELG.MA.HS.A.10: Solve systems of equations.** ○ **A-REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. ○ **A-REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Note: students solve 2x2 systems.
Algebra 2	**ELG.MA.HS.A.10: Solve systems of equations.** ○ **A-REI.6** - Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. ○ **A-REI.7** - Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.* Note: students solve 2x2 and 3x3 systems.

Students will demonstrate command of the ELG by:

- Writing a system of two linear inequalities or two linear equations in two variables to model a mathematical or real-world situation, solving the system, and interpreting the results in terms of the situation context using appropriate academic language.
- Solving a system of two linear equations graphically, algebraically, or with a table.
- Determining the nature of the solutions (infinite, exactly one, or none).
- Checking the solution to a system by substituting into both equations.
- Writing a solution to a system as an ordered pair, with both an x-value and a y-value.

Vocabulary:

- elimination
- infinite solutions
- intersection
- linear equation
- no solution
- parallel
- slope
- substitution
- system of equations

Sample Instructional/Assessment Tasks:

1) **Standard(s):** ELG.MA.HS.A.10 – (A-REI.6)

Source: Illustrative Mathematics
https://www.illustrativemathematics.org/content-standards/HSA/REI/C/6/tasks/935

Item Prompt:

A type of pasta is made of a blend of quinoa and corn. How much of the pasta is quinoa and how much is corn? The pasta company is not disclosing the percentage of each ingredient in the blend. However, quinoa contains a higher percentage of protein than corn. At the company's website, they give some information about the protein content of different foods and their own pasta blend, see http://www.quinoa.net/199.html. Use the protein content of each ingredient to find out how much quinoa and how much corn is in one serving of the pasta.

Correct Answer:

The information on the website states in the table titled "Quinoa's Food Value" that quinoa contains 16.2% protein and corn contains 3.5% of protein. The table at the bottom of the page titled "Gluten Free Pasta Nutrition Facts" lists that the pasta blend contains 4 grams of protein in a serving of size 57 grams. This is enough information to solve the problem with a system of two equations in two unknowns. If we let q be the amount of quinoa, in grams, in one serving of pasta and c be the amount of corn, in grams, in one serving of pasta we have $q + c = 57$. We also know that 16.2% of quinoa is protein and 3.5% of corn is protein and one serving of pasta contains 4 grams of protein. We can summarize this information in the equation $0.162q + 0.035c = 4$. Therefore, we have the following system of equations:

$$q + c = 57$$
$$0.162q + 0.035c = 4$$

We can solve this system using the method of substitution or the method of elimination. Using the method of substitution, we solve the first equation for q:

$$q = 57 - c.$$

We substitute this for q in the second equation and solve for c:

$$0.162(57 - c) + 0.035c = 4$$
$$9.234 - 0.162c + 0.035c = 4$$
$$9.234 - 0.127c = 4$$
$$-0.127c = -5.234$$
$$c = 41.213$$

So we have $c = 41$ and $q + c = 57$, therefore $q = 16$. Out of the 57 grams of pasta in one serving, 41 grams are corn and 16 grams are quinoa. In other words, about 72% of the pasta blend is corn and 28% is quinoa.

Note: In our solution to the problem, we are making the assumption that the protein content of the corn and the quinoa used to make the pasta is the same as the protein content listed in the table. This may or may not be the case. For example, Wikipedia lists the protein content of quinoa as 14% and of corn as 3.22%. Using these numbers, our solution would be different: 35% quinoa, 65% corn.

2) Standard(s): ELG.MA.HS.A.10 – (A-REI.6 and A-CED.A)

Source: Illustrated Mathematics
https://www.illustrativemathematics.org/content-standards/HSA/REI/C/6/tasks/462)

Item Prompt:

Nola was selling tickets at the high school dance. At the end of the evening, she picked up the cash box and noticed a dollar lying on the floor next to it. She said,
I wonder whether the dollar belongs inside the cash box or not.
The price of tickets for the dance was 1 ticket for \$5 (for individuals) or 2 tickets for \$8 (for couples). She looked inside the cash box and found \$200 and ticket stubs for the 47 students in attendance. Does the dollar belong inside the cash box or not?

Correct Answer:

Let s be the number of tickets sold to individuals and c be the number of tickets sold to couples. We know that 47 tickets were sold so far, so we have $s + c = 47$. Since each individual's ticket is \$5, the total amount of money made by selling tickets to individuals is $5s$. Similarly, since each ticket sold to couples is \$4, the total amount of money made by selling tickets to couples is $4c$. The cash box contains \$200 total, so we have $5s + 4c = 200$. Thus, we can represent the situation with a system of equations:

$$s + c = 47$$
$$5s + 4c = 200$$

Does this system of two equations in two unknowns have a positive integer solution? Furthermore, c has to be even, since tickets sold to couples were only sold in sets of 2. Solving the first equation for s we have

$$s = 47 - c.$$

Substituting for s in the second equation we obtain

$$5(47 - c) + 4c = 200$$

Distributing and collecting like terms we have

$$235 - c = 200$$

Therefore, $c = 35$ and $s = 12$. This means that without the found extra dollar in the cash box, 35 tickets were sold to couples and 12 tickets were sold to individuals. This is not possible, since couples tickets were only sold in sets of 2.
First indications are that the \$1 on the floor belongs in the cash box. To check this, we change the system of equations so that it reflects the extra \$1 in the cash box:

$$s + c = 47$$
$$5s + 4c = 201$$

The solution to the changed system of equations is $s = 13$ and $c = 34$, so 13 tickets were sold to individuals and 34 tickets were sold to couples. This combination of tickets is indeed possible.
Note that it is not really necessary to solve the second system of equations in the standard way. We could just have argued that exchanging one couples' ticket for an individual's ticket would increase the money in the cash box from 200 to 201 and it would result in an even number of couples tickets sold.

3) **Standard(s): HSA-REI.C.6**

Source: Illustrated Mathematics
https://www.illustrativemathematics.org/content-standards/HSA/REI/C/6/tasks/1833)

Item Prompt: Estimating a Solution via Graphs

Jason and Arianna are working on solving the equations

$$6x+17y=100$$
$$5x+9y=86$$

Rounding their answer to the nearest hundredth, Jason and Arianna find that $x \approx 4.04$ and $y \approx 7.31$.

- **a)** Explain, in terms of the slopes of the two linear equations, why you know that there is a unique solution to this system of two linear equations.
- **b)** Explain, in terms of the slopes and y-intercepts of the two linear equations, how you know Jason and Arianna have made a mistake in their calculations.
- **c)** Explain, looking at the graphs of these two equations, how you know Jason and Arianna have made a mistake in their calculations.

Correct Answer:

- **a)** The two equations provided are linear so their graphs will intersect in exactly one point provided they are not parallel. The slope of the line defined by the equation 6x+17y=100 is $\frac{-6}{17}$ while the slope of the line defined by the equation 5x+9y=86 is $\frac{-5}{9}$. Since $\frac{-6}{17} \neq \frac{-5}{9}$ these lines are not parallel and so they intersect in exactly one point.
- **b)** Note that the y intercept of the line defined by the equation 6x+17y=100 is $\frac{100}{17}$ or about 6. The y-intercept of the line defined by the equation 5x+9y=86 is $\frac{86}{9}$ or a little more than 9. The slope of this second line, $\frac{-5}{9}$, is less than the slope of the first, $\frac{-6}{17}$. This means that the lines will meet when x>0. An x value near 4 is far too small, however, because the y intercepts differ by more than 3 and the slopes differ by a very small amount (about 0.2). So x will need to be larger than 15 before the blue line drops enough, from the y-axis, to meet the blue line.

- **c)** The graphs of the two linear equations are shown below. They appear to meet when x is around 17 or 18 and y is about -1. This is very far from what Jason and Arianna found and so their answer is not reasonable.