Algebra 1

ELG HS.A.2: Write expressions in equivalent forms to solve problems

Vertical Progression:

7th Grade	**7.EE.A Use properties of operations to generate equivalent expressions.** ○ **7.EE.A.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. ○ **7.EE.A.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*
8th Grade	**8.EE.A Work with radicals and integer exponents.** ○ **8.EE.A.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
Algebra 1	**ELG.MA.HS.A.2 Write expressions in equivalent forms to solve problems.** ○ **A-SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ ○ **A-SSE.3a** Factor a quadratic expression to reveal the zeros of the function it defines. ○ **A-SSE.3.b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. ○ **A-SSE.3.c** Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%* [exponential expressions with integer exponents]
Algebra 2	**ELG.MA.HS.A.2 Write expressions in equivalent forms to solve problems.** ○ **A-SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ ○ **A-SSE.3.c** Use the properties of exponents to transform expressions for exponential functions. [exponential expressions with integer and rational or real exponents] ○ **A-SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

Students will demonstrate command of the ELG by:

- Factoring quadratic expressions to find zeros.
- Completing the square in a quadratic expression to find the maximum or minimum (vertex).
- Using properties of exponents to write equivalent exponential expressions.

Vocabulary:

- Equivalent
- Quadratic
- Zeros
- Vertex

Sample Instructional/Assessment Tasks:

1) **Standard(s): A-SSE.B.3**

Company Profit

Source: Illustrative Mathematics
https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/3/tasks/434

Item Prompt:

The profit that a company makes selling an item (in thousands of dollars) depends on the price of the item (in dollars). If *p* is the price of the item, then three equivalent forms for the profit are:
Standard form: $-2p^2 + 24p - 54$
Factored form: $-2(p-3)(p-9)$
Vertex form: $-2(p-6)^2 + 18$
Explain which form is most useful for finding

a. The prices that give a profit of zero dollars?
b. The profit when the price is zero?
c. The price that gives the maximum profit?

Correct Answer:

a. The factored form gives the values of p that make the profit zero. Since factored form is $-2(p-3)(p-9)$, the profit is zero when p=3 or p=9. The company breaks even if the price charged for the product is \$3 or \$9.
b. The standard form is the easiest one to use to find the profit when the price is zero. Substituting p=0 into the standard form $-2p^2 + 24p - 54$, we see that the profit is −54 (in thousands of dollars) when the price is zero. If the company gives the product away for free, it loses \$54,000.
c. The vertex form shows us what price maximizes profit. From the expression $-2(p-6)^2 + 18$, we see that the maximum profit is 18 thousand dollars, and it occurs when p=6. The company should charge a price of \$6 for this product.

2) **Standard(s): A-SSE.B.3**

Graphing Quadratics

Source: Illustrative Mathematics
https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/3/tasks/388

Item Prompt:

a. Graph these equations on your graphing calculator at the same time. What happens? Why?

$$y_1 = (x-3)(x+1)$$
$$y_2 = x^2 - 2x - 3$$
$$y_3 = (x-1)^2 - 4$$
$$y_4 = x^2 - 2x + 1$$

b. Below are the first three equations from the previous problem.

$$y_1 = (x-3)(x+1)$$
$$y_2 = x^2 - 2x - 3$$
$$y_3 = (x-1)^2 - 4$$

These three equations all describe the same function. What are the coordinates of the following points on the graph of the function? From which equation is each point most easily determined? Explain.

i. vertex: _____
ii. y-intercept: _____
iii. x-intercept(s): _____

c. Make up an equation for a quadratic function whose graph satisfies the given condition. Use whatever form is most convenient.

i. Has a vertex at (−2,−5).
ii. Has a y-intercept at (0,6)
iii. Has x-intercepts at (3,0) and (5,0)
iv. Has x-intercepts at the origin and (4,0)
v. Goes through the points (4,2) and (1,2)

Correct Answer:

a. When you graph these four equations, only two different parabolas are shown. This is because the first three equations are equivalent, and so all produce the same graph. We can see the equivalence as follows:

- If we multiply the factors given in the first equation, we'll get the second equation.
- Similarly, if we multiply out the perfect square and combine like terms in the third equation, we also get the second one.
- The fourth function produces a different graph. We can see that the difference between it and y_2 is just 4, so that graph is 4 units below the other one.

b. i. The vertex is (1,−4) which is most visible in y_3 since the vertex occurs at the point where the squared portion is zero.

ii. The y-intercept is (0,−3), which is visible as the constant in y_2 since the other terms are 0 when you plug in x=0.

iii. The x-intercepts are (3,0) and (−1,0), which are most visible in y_1 since you can find the roots of the polynomial using the zero factor property and thus the intercepts correspond to the zeros of each factor.

c. Each of these problems has many possible answers. Asking students for more possible answers is a great extension for students - it gets them thinking about the effects of the different parts of the equation.