Algebra 1

ELG HS.F.2: Interpret functions that arise in applications in terms of the context.

Vertical Progression:

7th Grade	**Use properties of operations to generate equivalent expressions.** **7.EE.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. **Analyze proportional relationships and use them to solve real-world and mathematical problems.** **7.RP.2** Recognize and represent proportional relationships between quantities. a) Decide whether two quantities are in a proportional relationship. b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c) Represent proportional relationships by equations. d) d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
8th Grade	**Use functions to model relationships between quantities.** ○ **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. ○ **8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
Algebra 1	**ELG.MA.HS.F.2 Interpret functions that arise in applications in terms of the context** ○ **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ ○ **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ ○ **F-IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ Note: Functions may include linear, quadratic, exponential, polynomial (quadratic or cubic), square root, cube root, and piecewise-defined functions (including step and absolute value).
Algebra 2	**ELG.MA.HS.F.2 Interpret functions that arise in applications in terms of the context** ○ **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ ○ **F-IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ Note: Functions may include linear, quadratic, exponential, polynomial, square root, cube root, piecewise defined (including step and absolute value), **rational, trigonometric, and logarithmic.**

Students will demonstrate command of the ELG by:

- Interpreting the key features of functions represented by graphs and tables in terms of the context.
- Sketching graphs showing key features given a verbal description of a relationship between quantities.
- Determining the domain for functions and relating the appropriate domain to the quantitative relationship it describes.
- Calculating and interpreting the average rate of functions (presented symbolically or as a table) over a specified interval, and estimating the rate of change from a graph.

Note: functions include linear, quadratic, exponential, square root, cube root, and piece-wise.

Note: key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

Vocabulary:

- Decreasing interval
- Domain
- End-behavior
- Function
- Increasing interval
- Intercept
- Interval
- Rate of change
- Relative maximum
- Relative minimum
- Slope
- Symmetry
- X-intercept
- Y-intercept

Sample Instructional/Assessment Tasks:

1) **Standard(s): ELG.MA.HS.F.2**

Summative Task for the conclusion of Unit 8

Source: Illustrated Mathematics
https://www.illustrativemathematics.org/content-standards/HSF/IF/B/tasks/578

Item Prompt:
In order to gain popularity among students, a new pizza place near school plans to offer a special promotion. The cost of a large pizza (in dollars) at the pizza place as a function of time (measured in days since February 10th) may be described as

$$C(t) = \begin{cases} 9, & 0 \le t \le 3 \\ 9 + t, & 3 < t \le 8 \\ 20, & 8 < t \le 28 \end{cases}$$

(Assume t only takes whole number values.)

a. If you want to give their pizza a try, on what date(s) should you buy a large pizza in order to get the best price?

b. How much will a large pizza cost on Feb. 18th?

c. On what date, if any, will a large pizza cost 13 dollars?

d. Write an expression that describes the sentence "The cost of a large pizza is at least *A* dollars *B* days into the promotion," using function notation and mathematical symbols only.

e. Calculate $C(9) - C(8)$ and interpret its meaning in the context of the problem.

f. On average, the cost of a large pizza goes up about 85 cents per day during the first two weeks of the promotion period. Which of the following equations best describes this statement?

1. $\frac{C(13)+C(0)}{2} = 0.85$
2. $\frac{C(13)-C(0)}{13} = 0.85$
3. $\frac{C(13)}{13} = 0.85$
4. $\frac{C(Feb.23)-C(Feb.10)}{13} = 0.85$

Correct Answer:

Based on the function above, the lowest price that the promotion offers for a large pizza is 9 dollars. This is the cost of the pizza when t = 0, t = 1, t = 2, and t = 3. We know that t denotes the number of days since February 10th. Thus, t = 0 corresponds to February 10th, t = 1 corresponds to February 11th, t = 2 corresponds to February 12th, and t = 3 corresponds to February 13th. Therefore, the best days to give the new pizza place a try in order to get the best price are February 10, February 11, February 12 and February 13.

a. February 18th is eight days after February 10th, corresponding to t = 8. Based on the function above, the cost of a large pizza in dollars is given by C(t) = 9 + t when 3 < t ≤ 8. Then, when t = 8, C(t) = 9 + 8 = 17 dollars. Thus, the cost of a large pizza on February 18th is 17 dollars.

b. We know that 13 ≠ 9 which implies that a large pizza cannot cost 13 dollars when 0 ≤ t ≤3 because for these values of t, C(t) = 9 based on the function above. Similarly, we know that 13 ≠ 20, which implies that a large pizza cannot cost 13 dollars when 8 < t < 28 because C(t) = 20 for these values. Thus, we know that the only time a large pizza could cost 13 dollars is when t is in the interval 3 < t ≤ 8, for which C(t) = 9 + t. In order to find out which date a large pizza will cost 13 dollars we must plug 13 into this equation as our cost and solve for t: $13 = 9 + t, t = 4$

This means that a large pizza costs 13 dollars when t = 4. We know that t denotes the days since February 10th so t = 4 corresponds to February 14th. Thus, a large pizza will cost 13 dollars on February 14th.

c. The statement that a pizza is at least A dollars B days into the promotion means that the cost of a large pizza B days into the promotion, denoted C(B), is greater than or equal to A dollars. Thus, an expression that describes this sentence using function notation and mathematical symbols is simply

$$C(B) \geq A$$

d. C(9) = 20 and C(8) = 9+8 = 17 based on the equation above. Thus, C(9) − C(8) = 20 – 17 = 3. C(9) = 20 corresponds to the cost of a large pizza 9 days after February 10th, or February 19th. C(8) = 17 corresponds to the cost of a large pizza 8 days after February 10th, or February 18th. Thus, the meaning of C(9) − C(8) = 3 in the context of the problem is that on February 19th, a large pizza will be 3 dollars more expensive than a large pizza on February 18th.

e. The first two weeks of the promotion take place from February 10th, when t = 0, to February 23rd, when t = 13. The average rate of change is given by:

$$\frac{C(13) - C(0)}{13 - 0} = \frac{20 - 9}{13} \approx 0.85$$

The expression above shows that the cost of a large pizza goes up about 0.85 dollars, or 85 cents, per day during the first two weeks of the promotion period. Therefore, the expression that best describes this statement is:

$$\frac{C(13) - C(0)}{13}$$

2) **Standard(s): ELG.MA.HS.F.2**

Source: Illustrated Mathematics
https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/2083

Item Prompt:

In this activity we will investigate the relationship between different quantities that are shown in a series of 10 video clips (see link above). Each clip includes a real time part, half speed playback, and a possible solution. (This allows students to make an initial sketch, adjust their sketch, and validate their results.)

For each video clip, watch the first part showing the situation in real time and then stop. At that point, sketch a graph of the relationship demonstrated on the set of graphs provided. In each graph pay special attention to key features such as increasing and decreasing intervals, maximums and minimums, intercepts, and constant and variable rates of change.

After drawing the initial sketch, watch the second part of the video showing the situation in half speed to double check your graph and revise it as appropriate.

Compare and discuss your graph with a neighbor or a different group. When you have reached an agreement of what the graph should look like, watch the last part of the video to see a possible solution. Did it agree with your graph? What was different, what was similar?

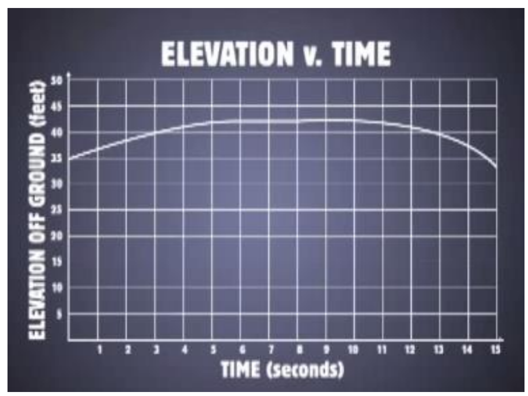
A handout and more videos are available at http://graphingstories.com/

Correct Answer:

Correct responses are included at the end of each video.

To emphasize the need to interpret the situation from the graph, draw all discussion and arguments from the graphs, not the videos. Students should justify reasoning based on the graph, not the video clip.

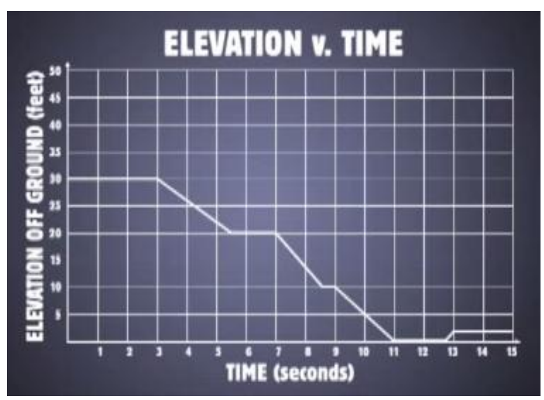
Video #1- Man walking over an arched bridge, pausing at the top to look over the rail. https://drive.google.com/file/d/0B6AE_k7mJ0JILWMwWTkwV0pYb0k



Discussion Questions:

- Why does the graph start and end so far above the time axis?
- At what time and for how long does the man stop to look over the rail? How does your graph reflect that?
- Where is the slope of the graph positive/negative? Why?
- When does he reach the highest point on the bridge? How do you know?

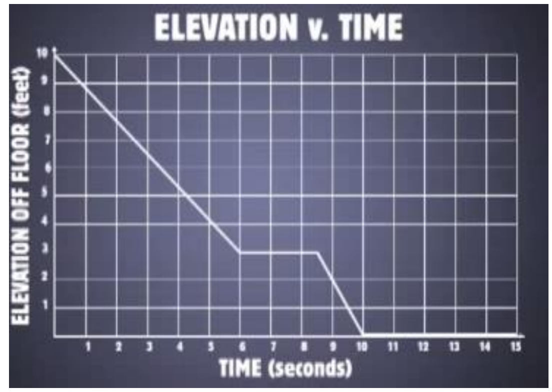
Video #2- Man walking down three flights of stairs with a landing between each, then stepping up onto a sidewalk.
https://drive.google.com/file/d/0B6AE_k7mJ0JIX0Qzc1ZSYk9EbEk



Discussion Questions:

- Where is the slope of the graph positive/negative/neither? What does a negative slope represent in the video? What does a positive slope represent in the video?

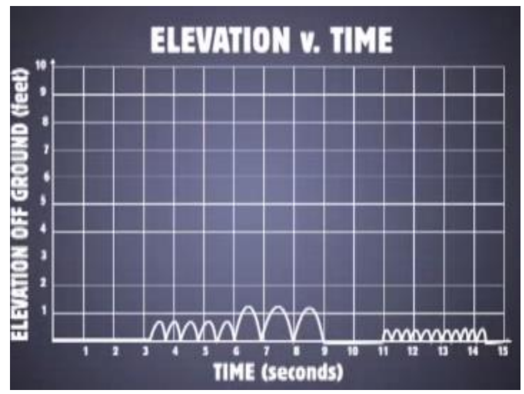
Video #3- Man coming down a ladder out of an attic, pausing to take a drink of soda, then walking into a kitchen.
https://drive.google.com/file/d/0B6AE_k7mJ0JIc1RFbGdZOHhuQWc



Discussion Questions:

- Because of the similarity between this video and video #2, this video could be used as an opportunity for students to check for understanding. In pairs, have students describe their graphs to each other in the context of the video.

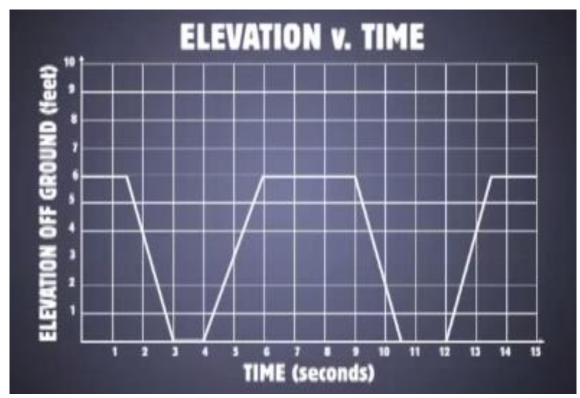
Video #4- Man hopping up and down to different heights and at different speeds.
https://drive.google.com/file/d/0B6AE_k7mJ0JIZm1FamtyeW9hRUE



Discussion questions:

- How is this graph similar to the previous 3? How is it different?
- When does the man jump the highest?
- How does the "shape" of his jumps change when he jumps higher or lower?
- When does he jump the highest? How do you know?

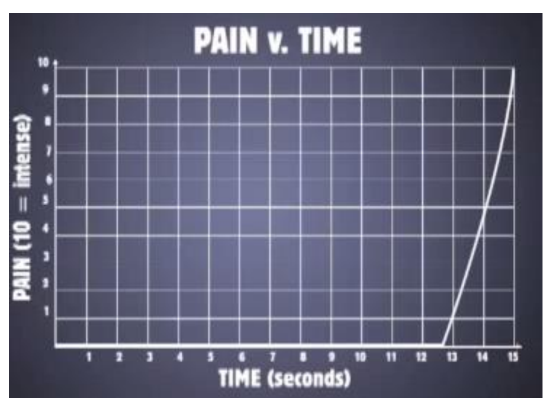
Video #5- Man going down one set of steps then up another set twice.
https://drive.google.com/file/d/0B6AE_k7mJ0JIMzF4dGZId2s3WTg



Discussion Questions:

- How long does it take the man to do one "lap" (down one side and back up the other)?
- How does the graph of his first lap compare to that of his second lap?
- How high is he when the video starts? How high when it ends? How do you know?

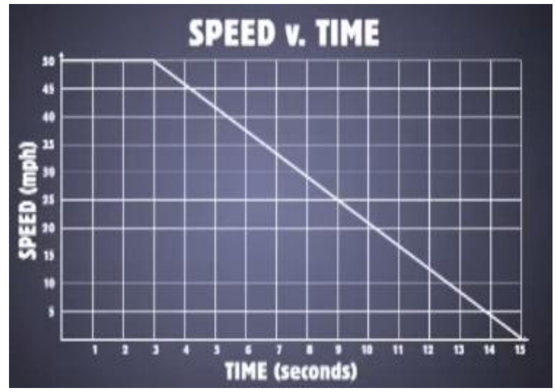
Video #6- Man hammering in nails, then hitting his thumb.
https://drive.google.com/file/d/0B6AE_k7mJ0JIZUN4VkxVbVQ4alk



Discussion Questions:

- Good place to discuss units. What units could we use to measure pain? Are there set units for pain? What do doctors use?
- What does the end of the graph look like?
- What might the graph look like if he hit his thumb earlier, then put it in ice?

Video #7- Speedometer of a car as the driver applies the brakes.
https://drive.google.com/file/d/0B6AE_k7mJ0JIVzFqQzFOOUdMLWs

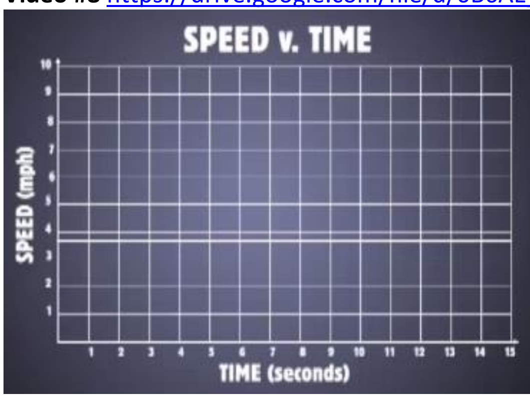


Discussion Questions:

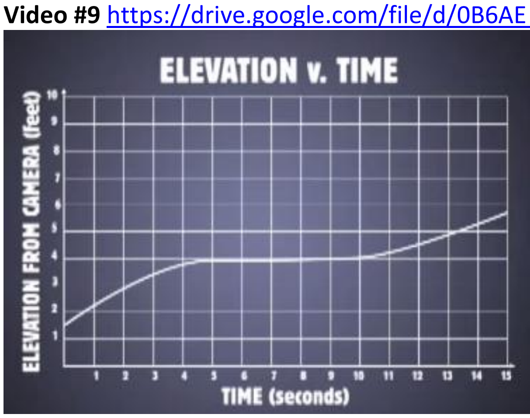
- When does the driver begin braking? (Use your graph)

Videos 8, 9, and 10 are the same clip of a man running up a curving road.

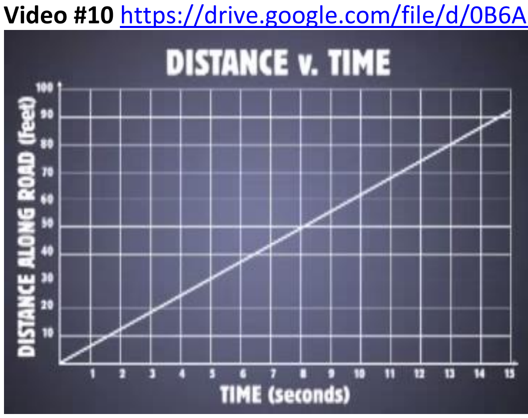
Video #8 https://drive.google.com/file/d/0B6AE_k7mJ0JIWEtEZlA0cVZpZFE



Video #9 https://drive.google.com/file/d/0B6AE_k7mJ0JIUEJoZi1FYzBsSms



Video #10 https://drive.google.com/file/d/0B6AE_k7mJ0JIWmNrNnlSLWxFbFk



Discussion Questions:

- How are these three graphs the same? How are they different?
- When is the runner running the fastest? When is he running slowest?
- When has he run halfway up the hill?
- What other graphs could we make using this same video?