Algebra 1

ELG HS.F.5: Build new functions from existing functions.

Vertical Progression:

7th Grade	**7.G: Draw, construct, and describe geometrical figures and describe the relationships between them.** ○ **7.G.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
8th Grade	**8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.** ○ **8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
Algebra 1	**ELG.MA.HS.F.5 Build new functions from existing functions.** ○ **F-BF.3** Identify effects on graphs of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find values of k given the graphs. Experiment with cases and illustrate explanations of effects on graphs using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (F-BF.3) Note: Functions may include linear, quadratic, exponential, polynomial (quadratic or cubic), square root, cube root, and piecewise-defined functions (including step and absolute value).
Algebra 2	**ELG.MA.HS.F.5 Build new functions from existing functions.** ○ **F-BF.3** Identify effects on graphs of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find values of k given the graphs. Experiment with cases and illustrate explanations of effects on graphs using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (F-BF.3) ○ **F-BF.4** Find inverse functions. ○ **F-BF.4a** Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$. Note: Functions may include linear, quadratic, exponential, polynomial, square root, cube root, piecewise defined (including step and absolute value), **rational, trigonometric, and logarithmic.**
	○ **F-BF.4b** (+) Verify by composition that one function is the inverse of another. ○ **F-BF.4c** (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. ○ **F-BF.4d** (+) Produce an invertible function from a non-invertible function by restricting the domain. ○ **F-BF.5** (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Students will demonstrate command of the ELG by:

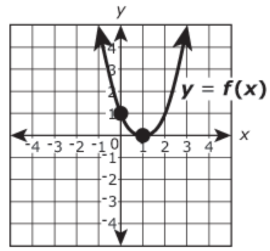
- Graph/identify vertical stretches and vertical shrinks (dilations)
- Graph/identify a horizontal stretches and horizontal shrinks (dilations).
- Graph/identify vertical and horizontal translations.
- Graph/identify vertical and horizontal reflections.
- Describe and explain the transformation of the parent function.

Vocabulary:

- horizontal shrink/stretch (dilation)
- horizontal translation
- parent function
- reflection
- transformation
- vertical shrink/stretch (dilation)
- vertical translation

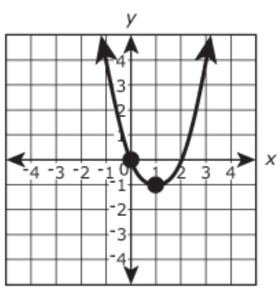
Sample Instructional/Assessment Tasks:

1) **Standard(s): F-BF.3**
Source: PARCC Algebra 1 Practice Test (EOY)
Item Prompt: Consider the function, $f(x)$, shown on the coordinate plane.



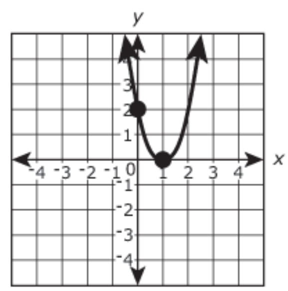
Identify the equations in the form $y = pf(x + r) + n$ which generate each of the graphs shown as a transformation of $f(x)$. Enter a number into **each** of the available boxes.

Part A



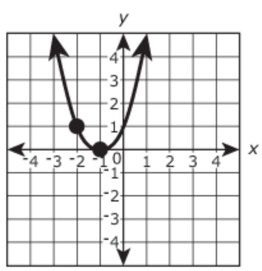
$y = \square f(x + \square) + \square$

Part B



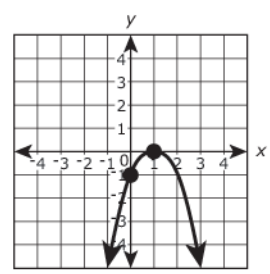
$y = \square f(x + \square) + \square$

Part C



$y = \square f(x + \square) + \square$

Part D



$y = \square f(x + \square) + \square$

Correct Answer:

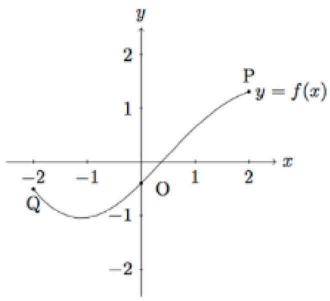
Part A: $y = 1\,f(x+0) + -1$
Part B: $y = 2\,f(x+0) + 0$
Part C: $y = 1\,f(x+2) + 0$
Part D: $y = -1f(x+0) + 0$

2) **Standard(s): F-BF.3**

Source: Illustrative Mathematics
https://www.illustrativemathematics.org/content-standards/HSF/BF/B/3/tasks/742

Item Prompt:

The figure shows the graph of a function *f* whose domain is the interval $-2 \le x \le 2$.



a. In (i)-(iii), sketch the graph of the given function and compare with the graph of *f*. Explain what you see.

i. $g(x) = f(x) + 2$
ii. $h(x) = -f(x)$
iii. $p(x) = f(x + 2)$

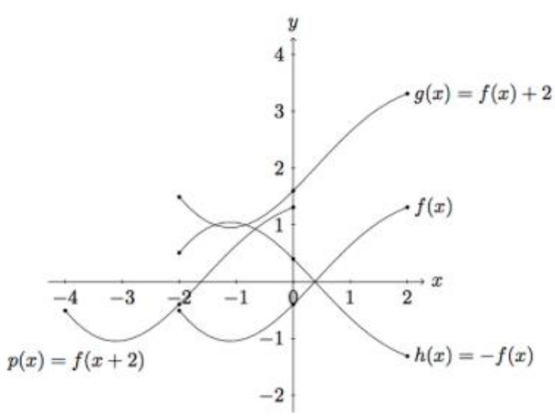
b. The points labelled *Q, O, P* on the graph of *f* have coordinates

Q = (–2, –0.509), *O* = (0, –0.4), *P* = (2, 1.309)

What are the coordinates of the points corresponding to *P, O, Q* on the graphs of *g, h,* and *p*?

Correct Answers:

a. The graphs of *f, g, h,* and *p* are shown (within the range of $-2 \le x \le 2$) below together to reveal their geometric relationship:



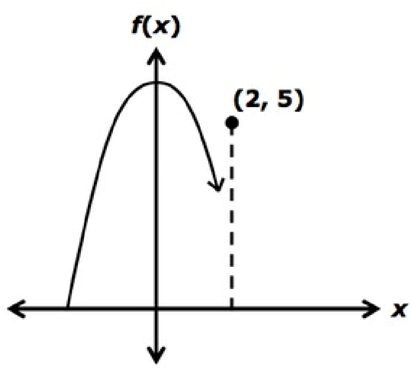
b. Q_g(-2, 1.491), Q_g(0, 1.6), P_g(2, 3.309)
Q_h(-2, 0.509), Q_h(0, 0.4), P_h(2, -1.309)
Q_p(-4, -0.509), Q_p(-2, -0.4), P_p(0, 1.309)

3) **Standard(s): F-BF.3**
Source: Adapted from Illustrative Mathematics
https://www.illustrativemathematics.org/content-standards/HSF/BF/B/3/tasks/695

Item Prompt: Medieval Archers

A computer game uses functions to simulate the paths of an archer's arrows. The x-axis represents the level ground on which the archer stands, and the coordinate pair (2,5) represents the top of a castle wall over which he is trying to fire an arrow.

In response to user input, the first arrow followed a path defined by the function $f(x) = 6 - x^2$, failing to clear the castle wall.

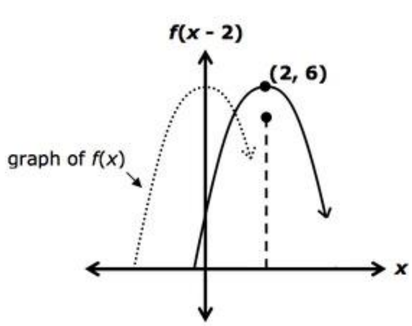


The next arrow must be launched with the same force and trajectory, so the user must reposition the archer in order for his next arrow to have any chance of clearing the wall.

a. How much closer to the wall must the archer stand in order for the arrow to clear the wall by the greatest possible distance?
b. What function must the user enter in order to accomplish this?
c. If the user didn't want to move the archer closer to the wall, how else could he/she modify the function to clear the wall?

Correct Answers:

a. The maximum value of f(x) is 6, which occurs at x=0. The maximum value must occur at x=2 in order for the arrow to clear the castle wall by the greatest margin. This is accomplished by moving the entire graph two units to the right, essentially moving the archer two units closer to the castle wall.



b. The user must enter a function whose output is f(0) when the input is x=2. A function that subtracts 2 from every input value before following the procedures of f would accomplish this--namely, f(x−2).
Since f is defined as f(x)=6−x2, the function f(x−2) is defined as follows:
$f(x-2) = 6-(x-2)^2 = 6-(x^2-4x+4) = 6-x^2+4x-4 = -x^2+4x+2$
c. Answer may vary. All correct answers should involve quadratics with a negative coefficient for x^2.