Algebra 1

ELG HS.F.6: Construct and compare linear, quadratic, and exponential models and solve problems.

Vertical Progression:

7th Grade	**Analyze proportional relationships and use them to solve real-world and mathematical problems.** ○ **7.RP.2** Recognize and represent proportional relationships between quantities. ○ **7.RP.2.a** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. ○ **7.RP.2.b** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. ○ **7.RP.2.c** Represent proportional relationships by equations. ○ **7.RP.2.d** Explain what a point (*x, y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.
8th Grade	**8.F.B Use functions to model relationships between quantities.** ○ **8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
Algebra 1	**ELG.MA.HS.F.6: Construct and compare linear, quadratic, and exponential models and solve problems.** ○ **F-LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions. ○ **F-LE.1a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. ○ **F-LE.1b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. ○ **F-LE.1c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. ○ **F-LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table). ○ **F-LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
Algebra 2	**ELG.MA.HS.F.6: Construct and compare linear, quadratic, and exponential models and solve problems.** ○ **F-LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table). ○ **F-LE.4** For exponential models, express as a logarithm the solution to $ab^{ct}=d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Students will demonstrate command of the ELG by:

- Determining whether a linear or exponential model would be more appropriate to model a given situation.
- Show in a graph or table where an exponentially increasing quantity exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- Constructing linear functions or arithmetic sequences given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- Constructing exponential functions or geometric sequences a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- Applying linear and exponential functions to real-world applications.
- Applying arithmetic and geometric sequences to real-world applications.

Vocabulary:

- arithmetic sequence
- exponential function
- exponential growth/decay
- geometric sequence
- linear function
- polynomial function

Sample Instructional/Assessment Tasks:

1) **Standard(s) F-LE.A.1 , F-LE.A.2**

Choosing an appropriate growth model

Source: Illustrated Mathematics
https://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/1594)

Item prompt:

Below are population estimates for the larger metropolitan areas of Paris (France), Shenzhen (China), and Lagos (Nigeria) for each decade between 1950 and 2010:

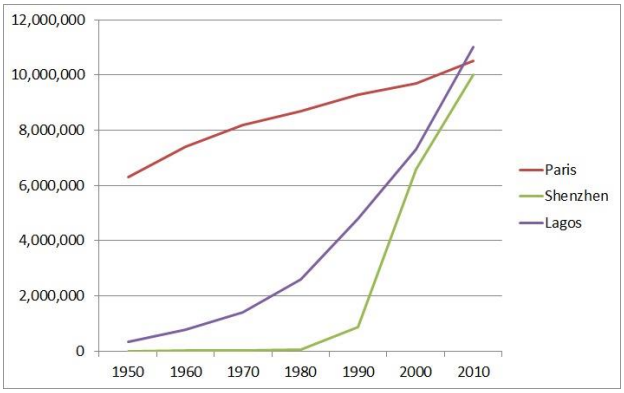
City	1950	1960	1970	1980	1990	2000	2010
Paris	6,300,000	7,400,000	8,200,000	8,700,000	9,300,000	9,700,000	10,500,000
Shenzhen	3100	8000	22,000	58,000	875,000	6,600,000	10,000,000
Lagos	330,000	760,000	1,400,000	2,600,000	4,800,000	7,300,000	11,000,000

a. For each city, decide if the population data can be accurately modeled by a linear, quadratic, and/or exponential function. Explain.

b. If you found one or more good models for a city population, what predictions would those models make for future decades? Are these reasonable?

Correct Answers:

Answers may vary. To get an idea of which model might be appropriate, we have plotted the population data together on the graph below: in this graph, the data points for each city have been joined by line segments.



Visual examination reveals that the Paris population seems to be approximately linear, the Lagos population data appears to be modeled by a quadratic function, while the Shenzhen population is likely exponential though the rate of growth is slowing down after the turn of the 21st century.

2) Functions

Standard(s) F-LE.A.3

Source: Inside Mathematics
http://www.insidemathematics.org/assets/common-core-math-tasks/functions.pdf

Item Prompt:

On the grid are eight points from two different functions.

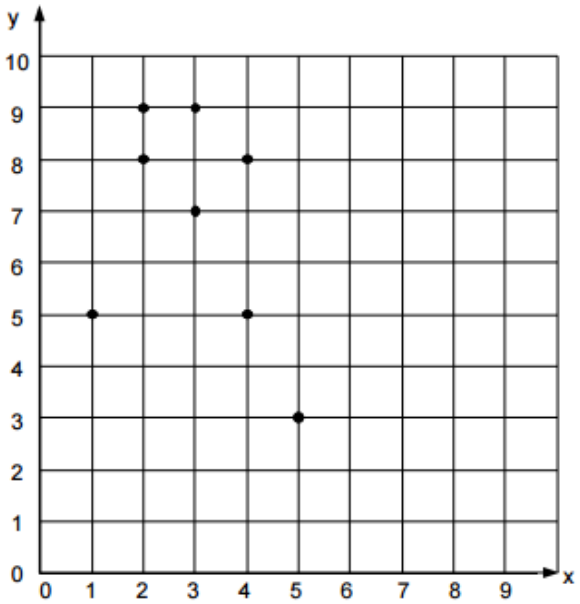
- four points fit a **linear** function
- the other four points fit a **non-linear** function.

a. For the **linear** function:

i. Write the coordinate pairs of its four points.
ii. Draw the line on the grid.
iii. Write an equation for the function.

b. For the **non-linear** function:

i. Write the coordinate pairs of its four points.
ii. Draw the graph of the function on the grid.
iii. Draw the function on the grid



c. Chris says: "The non-linear function is quadratic." Monique says: The non-linear function is exponential." Who is correct? Explain your reasons.

Correct Answers:

a. i. (2, 9), (3, 7), (4, 5), (5, 3)
   ii. Draws line on grid
   iii. $y = 13 - 2x$

b. i. (1, 5), (2, 8), (3, 9), (4, 8)
   ii. Draws line on grid

c. Chris. Explanations may vary.

Note: A more detailed explanation of correct responses and sample student responses can be found via the link above.