Algebra 1

ELG HS.N.2: Use properties of rational and irrational numbers

Vertical Progression:

7th Grade	**7.NS.A Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** ○ **7.NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. ○ **7.NS.A.1.d** Apply properties of operations as strategies to add and subtract rational numbers. ○ **7.NS.A.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. ○ **7.NS.A.2.a** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. ○ **7.NS.A.2.b** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts.
8th Grade	**8.NS.A Know that there are numbers that are not rational, and approximate them by rational numbers.** ○ **8.NS.A.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. ○ **8.NS.A.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).
Algebra 1	**Algebra 1: ELG.MA.HS.N.2 Use properties of rational and irrational numbers.** ○ **N-RN.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
Algebra 2	**Algebra 2: ELG.MA.HS.N.1 Extend the properties of exponents to rational exponents.** ○ **N-RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents ○ **N-RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Students will demonstrate command of the ELG by:

- Explaining why the sum or product of two rational numbers is rational.
- Explaining that the sum of a rational number and an irrational number is irrational.
- Explaining that the product of a nonzero rational number and an irrational number is irrational.

Vocabulary:

- irrational
- rational

Sample Instructional/Assessment Tasks:

1) **Standard(s): N-RN.3**

Source: Illustrated Mathematics
https://www.illustrativemathematics.org/content-standards/HSN/RN/B/3/tasks/690

Item Prompt: Operations with Rational and Irrational Numbers

Experiment with sums and products of two numbers from the following list to answer the questions that follow:

$$5\ ,\sqrt{2}\ ,\frac{1}{2}\ ,0\ ,-\sqrt{2}\ ,\frac{1}{\sqrt{2}}\ ,\pi$$

Based on the above information, conjecture which of the statements is ALWAYS true, which is SOMETIMES true, and which is NEVER true?

a. The sum of a rational number and a rational number is rational.
b. The sum of a rational number and an irrational number is irrational.
c. The sum of an irrational number and an irrational number is irrational.
d. The product of a rational number and a rational number is rational.
e. The product of a rational number and an irrational number is irrational.
f. The product of an irrational number and an irrational number is irrational.

Correct Answer:

a. ALWAYS true.
b. ALWAYS true.
c. SOMETIMES true (for instance, the sum of additive inverses like $\sqrt{2}$ and $-\sqrt{2}$ will be 0).
d. ALWAYS true.
e. SOMETIMES true (this holds except when the rational number is zero).
f. SOMETIMES true (for instance, the product of multiplicative inverses like $\sqrt{2}$ and $\frac{1}{\sqrt{2}}$ will be 1).

2) Standard(s): N-RN.3

Source: Illustrated Mathematics
https://www.illustrativemathematics.org/content-standards/HSN/RN/B/3/tasks/1817

Item Prompt: Sums of Rational and Irrational Numbers

Kaylee says:

I know that π is an irrational number so its decimal never repeats. I also know that $\frac{1}{7}$ is a rational number so its decimal repeats. But I don't know how to add or multiply these decimals so I am not sure if $\pi + \frac{1}{7}$ and $\pi \cdot \frac{1}{7}$ are rational or irrational.

Help Kaylee decide whether or not $\pi + \frac{1}{7}$ and $\pi \cdot \frac{1}{7}$ are rational or irrational.

Correct Answer:

The number $x = \pi + \frac{1}{7}$ is either rational or irrational. If x were rational then $x - \frac{1}{7}$ would also be rational (a rational ± a rational is rational). But $x - \frac{1}{7} = \pi$ which is not rational. Since x is not rational it must be irrational.

A similar argument shows that $x = \pi \cdot \frac{1}{7}$ is irrational. Again, x is either rational or irrational. If x were rational then 7x would also be rational but 7x = π which is irrational. So x must be irrational.