Vertical Progression:

7th Grade	**7.EE.A Use properties of operations to generate equivalent expressions.** o **7.EE.A.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
Algebra 1	**ELG.MA.HS.A.3 Perform arithmetic operations on polynomials.** o **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Algebra 2	**ELG.MA.HS.A.5 Use polynomial identities to solve problems.** o **A-APR.4** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
	ELG.MA.HS.A.5 Use polynomial identities to solve problems. o **A-APR.5 (+)**Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

Students will demonstrate command of the ELG by:

- Proving polynomial identities.
- Using polynomial identities to describe numerical relationships.

Vocabulary:

- numerical relationship
- polynomial identity

[1] The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument

Sample Instructional/Assessment Tasks:

1) **Standard(s): A-APR.4**
Source: Adapted from http://www.azed.gov/azcommoncore/mathstandards/hsmath/

Item Prompt:

Part A: Prove that $(n+1)^2 - n^2 = 2n+1$ for any whole number *n*.

Part B: Show that $(n+1)^2 - n^2$ can be used to generate odd numbers.

Correct Answer:

Part A: response expands the expression on the left side of the equation and combines like terms to generate an expression equal to the right side of the equation.

Part B: student substitutes values for *n* (e.g., 1, 2, 3) for the expression on the left side of the equation and evaluates expressions to generate odd numbers (e.g., 1, 3, 5).

2) **Standard(s): A-APR.4**

Source: **https://www.illustrativemathematics.org/content-standards/HSA/APR/C/4/tasks/594**

Item Prompt:

Alice was having a conversation with her friend Trina, who had a discovery to share:

"Pick any two integers. Look at the sum of their squares, the difference of their squares, and twice the product of the two integers you chose. Those three numbers are the sides of a right triangle."

Trina had tried this several times and found that it worked for every pair of integers she tried. However, she admitted that she wasn't sure whether this "trick" always works, or if there might be cases in which the trick doesn't work.

a. Investigate Trina's conjecture for several pairs of integers. Does her trick appear to work in all cases, or only in some cases?

b. If Trina's conjecture is true, then give a precise statement of the conjecture, using variables to represent the two chosen integers, and prove it. If the conjecture is not true, modify it so that it is a true statement, and prove the new statement.

c. Use Trina's trick to find an example of a right triangle in which all of the sides have integer length, all three sides are longer than 100 units, and the three side lengths do not have any common factors.

Correct Answer:

Part A

Student tries out values to show that Trina is correct

Part B

Suppose that m and n are positive integers such that $m > n$. Then the numbers $m^2 + n^2$, $m^2 - n^2$, and $2mn$ are the lengths of the sides of a right triangle.

To prove this, it suffices to prove that when these three numbers are squared, one square is the sum of the other two. In our table, it appeared that $m^2 + n^2$ was always the largest of the three numbers, so we conjecture that

$$(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2.$$

Expanding the left side, we obtain

$$(m^2 + n^2)^2 = m^4 + 2m^2n^2 + n^4.$$

Expanding the right side, we obtain

$$\begin{aligned}(m^2 - n^2)^2 + (2mn)^2 &= m^4 - 2m^2n^2 + n^4 + 4m^2n^2 \\ &= m^4 + 2m^2n^2 + n^4.\end{aligned}$$

Part C

To find a triangle satisfying the given requirements, we try using Trina's trick on the integers $m = 13$ and $n = 6$. We get $m^2 + n^2 = 205$, $m^2 - n^2 = 133$, and $2mn = 156$. By the reasoning given above, we know that there is a right triangle with sides of length 133, 156, and 205. Furthermore, the numbers 133, 156, and 205 have no common factors.