## Vertical Progression:

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| **7th Grade** | **7.EE.A Use properties of operations to generate equivalent expressions.**<br>o **7.EE.A.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. |
| **Algebra 1** | **ELG.MA.HS.A.3 Perform arithmetic operations on polynomials.**<br>o **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. |
| **Algebra 2** | **ELG.MA.HS.A.6 Rewrite rational expressions.**<br>o **A-APR.6** Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x),* where *a(x), b(x), q(x)*, and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x),* using inspection, long division, or, for the more complicated examples, a computer algebra system. |
| | **ELG.MA.HS. A.6 Rewrite rational expressions.**<br>o **A-APR.7 (+)**Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. |

## Students will demonstrate command of the ELG by:

- Rewriting simple rational expressions using inspection, long division, or a computer algebra system.

## Vocabulary:

- computer algebra system
- rational expression

## Sample Instructional/Assessment Tasks:

1) **Standard(s): A-APR.6**

**Source: Kansas Flip Book**

**Item Prompt:**

Find the quotient and remainder for the rational expression $\frac{x^3-3x^2+x-6}{x^2+2}$ and use them to write the expression in a different form.

**Correct Answer:**

**The quotient is $x-3$ and reminder is $-x$. One expression that could be written is** $x-3+\frac{-x}{x^3-3x^2+x-6}$

2) **Standard(s): A-APR.6**
**Source:** Adapted from https://www.illustrativemathematics.org/content-standards/HSA/APR/D/6/tasks/825

**Item Prompt:**

The US Department of Energy keeps track of fuel efficiency for all vehicles sold in the United States. Each car has two fuel economy numbers, one measuring efficient for city driving and one for highway driving. For example, a 2012 Volkswagen Jetta gets 29.0 miles per gallon (mpg) in the city and 39.0 mpg on the highway.

Many banks have "green car loans" where the interest rate is lowered for loans on cars with high combined fuel economy. This number is not the average of the city and highway economy values. Rather, the combined fuel economy (as defined by the federal Corporate Average Fuel Economy standard) for $x$ mpg in the city and $y$ mpg on the highway, is computed as

$$\text{combined fuel economy} = \frac{1}{\frac{1}{2}\left(\frac{1}{x}+\frac{1}{y}\right)}.$$

a. What is the combined fuel economy for the 2012 Volkswagen Jetta? Give your answer to three significant digits.

b. For most conventional cars, the highway fuel economy is 10 mpg higher than the city fuel economy. If we set the city fuel economy to be $x$ mpg for such a car, what is the combined fuel economy in terms of $x$? Write your answer as a single rational function, $a(x)/b(x)$.

c. Rewrite your answer from (b) in the form of $q(x)+\frac{r(x)}{b(x)}$ where $q(x)$, $r(x)$ and $b(x)$ are polynomials and the degree of $r(x)$ is less than the degree of $b(x)$.

d. Use your answer in (c) to conclude that if the city fuel economy, $x$, is large, then the combined fuel economy is approximately $x+5$.

**Correct Answer:**

At least by part (b), it becomes more convenient to re-write the expressions by

$$\frac{1}{\frac{1}{2}\left(\frac{1}{x}+\frac{1}{y}\right)} = \frac{2xy}{x+y}.$$

We'll do this at the start, though part (a) could certainly be done using the original form of the expression.

a. For $x = 29.0$ mpg and $y = 39.0$ mpg, compute that

$$\text{combined fuel economy} = \frac{2(29.0)(39.0)}{29.0+39.0} \approx 33.265.$$

To three significant digits, this is 33.3 mpg.

We note that this exercise is an opportunity to pay close attention to units, especially since the units of a harmonic mean of two quantities are not immediately obvious. Represent mpg as $\frac{\text{miles}}{\text{gallon}}$ the same computation done with units looks like

$$\frac{2\left(29.0\frac{\text{miles}}{\text{gallon}}\right)\left(39.0\frac{\text{miles}}{\text{gallon}}\right)}{29.0\frac{\text{miles}}{\text{gallon}}+39.0\frac{\text{miles}}{\text{gallon}}} \approx 33.265\frac{\left(\frac{\text{miles}}{\text{gallon}}\right)^2}{\frac{\text{miles}}{\text{gallon}}} = 33.265\frac{\text{miles}}{\text{gallon}}.$$

b. For $y = x + 10$, we have

$$\text{combined fuel economy} = \frac{2x(x+10)}{2x+10} = \frac{2x(x+10)}{2(x+5)} = \frac{x(x+10)}{(x+5)}.$$

c. A student might calculate this reduction using long division, synthetic division or grouping. For any method, we have

$$\frac{x(x+10)}{(x+5)} = x + 5 - \frac{25}{x+5}$$

d. When $x$ is large, $25/(x+5)$ is small. In particular, when $x > 20$, this term is less than 1 so the approximation of $x + 5$ is within 1 mpg of the correct value of the combined fuel economy.