

Vertical Progression:

8 th Grade	<p>8.EE.C Analyze and solve linear equations and pairs of simultaneous linear equations.</p> <ul style="list-style-type: none"> ○ 8.EE.C.7 Solve linear equations in one variable ○ 8.EE.C.7a Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers.) ○ 8.EE.C.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
Algebra 1	<p>ELG.MA.HS.A.9 Solve equations and inequalities in one variable</p> <ul style="list-style-type: none"> ○ A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. ○ A-REI.4 Solve quadratic equations in one variable. ○ A-REI.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. ○ A-REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. [Note: Algebra 1 students are only required to recognize that complex roots are not real numbers.]
Algebra 2	<p>ELG.MA.HS.A.9 Solve equations and inequalities in one variable.</p> <ul style="list-style-type: none"> ○ A-REI.4 Solve quadratic equations in one variable. ○ A-REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Students will demonstrate command of the ELG by:

- Solving quadratic equations by a variety of methods (inspection, square roots, factoring, completing the square, quadratic formula).
- Recognizing when the quadratic formula has complex roots and expressing those roots in the form $a \pm bi$ for real numbers a and b .

Vocabulary:

- complex root
- quadratic equation
- quadratic formula

Sample Instructional/Assessment Tasks:

1) Standard(s): A-REI.B.4

Source: <https://www.illustrativemathematics.org/content-standards/HSA/REI/A/2/tasks/1926>

Item Prompt:

Jamie and Ralph take a canoe trip up a river for 1 mile and then return. The current in the river is 1 mile per hour. The total trip time is 2 hours and 24 minutes. Assuming that they are paddling at a constant rate throughout the trip, find the speed that Jamie and Ralph are paddling.

Correct Answer:

Suppose we let x denote the speed, in miles per hour, that the canoe would travel with no current. When they are traveling against the current, Jamie and Ralph's speed will be $x-1$ miles per hour and when they are traveling with the current their speed will be $x+1$ miles per hour. The trip upstream will take $\frac{1}{x-1}$ hours and the trip downstream will take $\frac{1}{x+1}$ hours. There are $\frac{2}{5}$ of an hour in 24 minutes so the total trip lasts for $2\frac{2}{5}$ hours giving us:

$$\frac{1}{x-1} + \frac{1}{x+1} = \frac{12}{5}$$

Multiplying both sides of the equation by $(x-1)(x+1)=x^2-1$ gives

$$(x+1) + (x-1) = \frac{12}{5}(x^2-1)$$

This equation simplifies to $\frac{12}{5}x^2 - 2x - \frac{12}{5} = 0$ or, after multiplying by $\frac{5}{2}$,

$$6x^2 - 5x - 6 = 0$$

We can use the quadratic formula to solve for x :

$$x = \frac{5 \pm \sqrt{5^2 - 4(6)(-6)}}{2(6)}$$

We have $\sqrt{169} = 13$ so the two solutions are $x = \frac{5+13}{12}$ or $x = \frac{3}{2}$ and $x = -\frac{2}{3}$. The second solution does not make any sense in this context as the speed cannot be negative. So Jamie and Ralph are paddling at a rate of $\frac{3}{2}$ miles per hour. Going upstream, the trip takes longer against the current and going with the current the trip is shorter.

2) Standard(s): A-REI.4b, N-CN.7

Source: PARCC Practice PBA Test #10

Item Prompt:

What are the solutions to the equation $2x^2 - x + 1 = 0$?

Correct Answer:

$$\frac{1}{4} - \left(\frac{\sqrt{7}}{4}\right)i \text{ and } \frac{1}{4} + \left(\frac{\sqrt{7}}{4}\right)i$$