Vertical Progression:

8th Grade	**8.G.B Understand and apply the Pythagorean Theorem.** o **8.G.B.6** Explain a proof of the Pythagorean Theorem and its converse. o **8.G.B.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
Geometry	**ELG.MA.HS.G.6 Prove theorems involving similarity** o **G-SRT.4** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity*.
Algebra 2	**ELG.MA.HS.F.10 Prove and apply trigonometric identities.** o **F-TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
	ELG.MA.HS.F.10 Prove and apply trigonometric identities. o **F-TF.9 (+)**Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Students will demonstrate command of the ELG by:

- Solving for $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle using the Pythagorean identity $\sin^2(\theta)+\cos^2(\theta)=1$.

Vocabulary:

- Pythagorean identity
- trigonometric identity

Sample Instructional/Assessment Tasks:

1) **Standard(s): F-TF.10**
Source: https://www.illustrativemathematics.org/content-standards/HSF/TF/C/8/tasks/1835

Item Prompt:

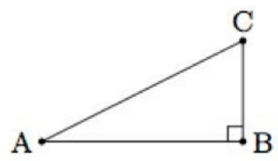
Suppose that $\cos\theta = \frac{2}{5}$ and that θ is in the 4th quadrant. Find $\sin\theta$ and $\tan\theta$ exactly.

Correct Answer:

$$\sin\theta = \frac{-\sqrt{21}}{5}, \ \tan\theta = \frac{-\sqrt{21}}{2}$$

2) **Standard(s): F-FT.10**
Source: https://www.illustrativemathematics.org/content-standards/HSF/TF/C/8/tasks/1693

Item Prompt:



a. In the triangle pictured above show that

$$\left(\frac{|AB|}{|AC|}\right)^2 + \left(\frac{|BC|}{|AC|}\right)^2 = 1$$

b. Deduce that $\sin^2\theta + \cos^2\theta = 1$ for any acute angle θ.

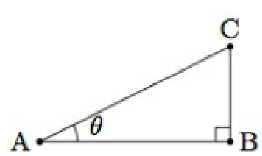
c. If θ is in the second quadrant and $\sin\theta = \frac{8}{17}$ what can you say about $\cos\theta$? Draw a picture and explain.

Correct Answers:

a. The Pythagorean Theorem says that if $\triangle ABC$ is a right triangle with right angle B then $|AB|^2 + |BC|^2 = |AC|^2$. Dividing both sides by $|AC|^2$ gives

$$\left(\frac{|AB|}{|AC|}\right)^2 + \left(\frac{|BC|}{|AC|}\right)^2 = 1.$$

b. If $0 < m(\theta) < 90$, then we can make a right triangle ABC, as pictured in the problem statement, so that $m(\angle BAC) = \theta$:

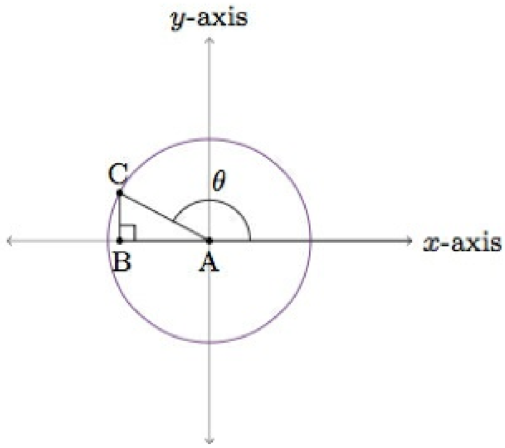


Then from part (a) we have

$$\left(\frac{|AB|}{|AC|}\right)^2 + \left(\frac{|BC|}{|AC|}\right)^2 = 1.$$

We also know that $\frac{|AB|}{|AC|} = \cos\theta$ and $\frac{|BC|}{|AC|} = \sin\theta$ so we have $\sin^2\theta + \cos^2\theta = 1$.

c. Below is a picture of an angle θ in the second quadrant with $\sin\theta = \frac{8}{17}$:



In the picture, the purple circle is the unit circle. The coordinates of C are $(\cos\theta, \sin\theta)$ and since C lies on the unit circle we have

$$\sin^2\theta + \cos^2\theta = 1.$$

Since $\sin\theta = \frac{8}{17}$ we can solve for $\cos\theta$ and we find $\cos\theta = \pm\frac{15}{17}$. Since we are in the second quadrant $\cos\theta = -\frac{15}{17}$.