Algebra 2

ELG HS.F.3: **Analyze functions using different representations.**

Vertical Progression:

8th Grade	**8.EE.B Understand the connections between proportional relationships, lines, and linear equations.** o **8. EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. **8.F.A Define, evaluate, and compare functions.** o **8.F.A.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
Algebra 1	**ELG.MA.HS.F.IF Analyze functions using different representations.** o **F-IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ o **F-IF.7a** Graph linear and quadratic functions and show intercepts, maxima, and minima. ★ o **F-IF.7b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ★ o **F-IF.7e** Graph exponential functions, showing intercepts and end behavior. ★ o **F-IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. o **F-IF.8a** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Note: Functions may include linear, quadratic, exponential, polynomial (quadratic or cubic), square root, cube root, and piecewise-defined functions (including step and absolute value).
Algebra 2	**ELG.MA.HS.F.3 Analyze functions using different representations.** o **F-IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ o **F-IF.7c** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. ★ o **F-IF.7e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ★ o **F-IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. o **F-IF.8b** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay*. o **F-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* Note: Functions may include linear, quadratic, exponential, polynomial, square root, cube root, piecewise defined (including step and absolute value), **rational, trigonometric, and logarithmic.**
	ELG.MA.HS.F.3 Analyze functions using different representations. o **F-IF.7d (+)**Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. ★

Students will demonstrate command of the ELG by:

- Graphing logarithmic, polynomial, and trigonometric functions expressed symbolically.
- Comparing the properties of two functions (logarithmic, polynomial, trigonometric, or rational) having different parent functions represented in different ways.
- Identifying domains and ranges of different functions.

Vocabulary:

- asymptotes
- domain
- end behavior
- exponential function
- logarithmic function
- maxima
- minima
- range
- trigonometric function

Sample Instructional/Assessment Tasks:

1) **Standard(s): F-IF.7c**

Source: https://www.illustrativemathematics.org/content-standards/HSF/IF/C/7/tasks/627

Item Prompt:

Graphs of Power functions:

a. Sketch the graphs of the functions described by $f(x) = x^2$ and $g(x) = x^4$ on the same axes, being careful to label any points of intersection. Also, find and label $\left(\frac{1}{2}, f(\frac{1}{2})\right)$ and $\left(\frac{1}{2}, g(\frac{1}{2})\right)$.

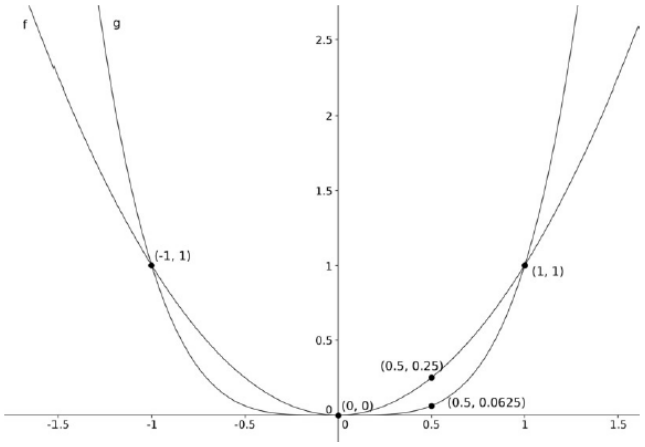
b. Sketch the graphs of the functions described by $f(x) = x^3$ and $g(x) = x^5$ on the same axes, being careful to label any points of intersection. Also, find and label $\left(\frac{1}{2}, f(\frac{1}{2})\right)$ and $\left(\frac{1}{2}, g(\frac{1}{2})\right)$.

c. Sketch the graphs of the functions described by $f(x) = x^2$ and $g(x) = x^3$ on the same axes, being careful to label any points of intersection. Also, find and label $\left(\frac{1}{2}, f(\frac{1}{2})\right)$ and $\left(\frac{1}{2}, g(\frac{1}{2})\right)$.

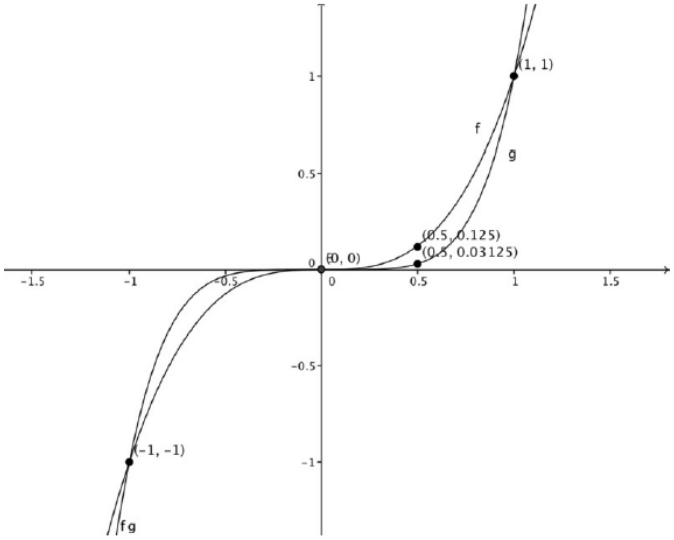
Correct Answesr:

The graphs are shown below.

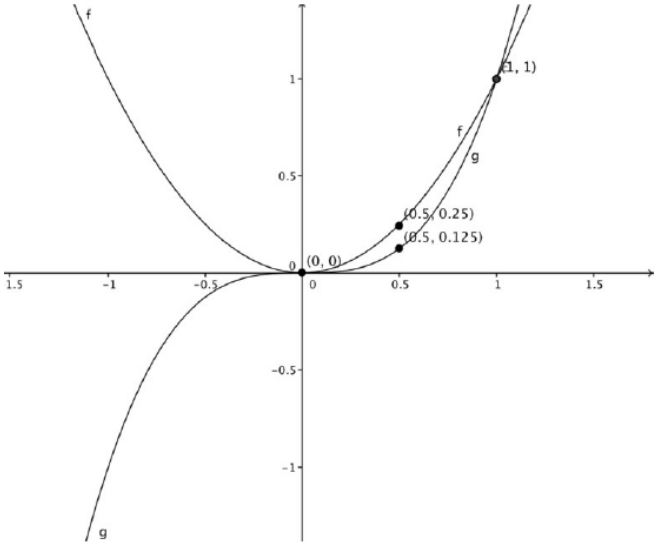
a.


f
g
(-1, 1)
(1, 1)
(0.5, 0.25)
(0, 0)
(0.5, 0.0625)

b.


(1, 1)
f
g
(0.5, 0.125)
(0.5, 0.03125)
(0, 0)
(-1, -1)
f g

c.


f
(1, 1)
f
g
(0.5, 0.25)
(0.5, 0.125)
(0, 0)
g

2) **Standard(s): F-IF.7**

Source: https://www.illustrativemathematics.org/content-standards/HSF/IF/C/7/tasks/1539

Item Prompt:

Many computer applications use very complex mathematical algorithms. The faster the algorithm, the more smoothly the programs run. The running time of an algorithm depenc the total number of steps needed to complete the algorithm. For image processing, the ru time of an algorithm increases as the size of the image increases.

For an n-by-n image, algorithm 1 has running time given by $p(n) = n^3 + 3n + 1$ and algori has running time given by $q(n) = 15n^2 + 5n + 4$ (measured in nanoseconds, or 10^{-9} seco

- Compute the running time for both algorithms for images of size 10-by-10 pixels and 10 100 pixels.
- Graph both running time polynomials in an appropriate window (or several windows if necessary).
- Which algorithm is more efficient?

Correct Answer:

a. For a 10-by-10 image, the two algorithms take respectively

$p(10) = 10^3 + 3(10) + 1 = 1031$ and $q(10) = 15(10^2) + 5(10) + 4 = 15$

nanoseconds.

For a 100-by-100 image, we have

$p(100) = 100^3 + 3(100) + 1 = 1{,}000{,}301$ and $q(100) = 15(100^2) + 5(100) + 4 =$

nanoseconds.

b. Graphs not shown.

c. For very small images, algorithm 1 has a shorter running time than algorithm 2. However, for any kind of realistic image size, algorithm 2 has much shorter running times than algorithm 1.