Vertical Progression:

8th Grade	**8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.** o **8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
Algebra 1	**ELG.MA.HS.F.5 Build new functions from existing functions.** o **F-BF.3** Identify effects on graphs of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find values of k given the graphs. Experiment with cases and illustrate explanations of effects on graphs using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Note: Functions may include linear, quadratic, exponential, polynomial (quadratic or cubic), square root, cube root, and piecewise-defined functions (including step and absolute value).
Algebra 2	**ELG.MA.HS.F.5 Build new functions from existing functions.** o **F-BF.3** Identify effects on graphs of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find values of k given the graphs. Experiment with cases and illustrate explanations of effects on graphs using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. o **F-BF.4** Find inverse functions. o **F-BF.4a** Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$. Note: Functions may include linear, quadratic, exponential, polynomial, square root, cube root, piecewise defined (including step and absolute value), **rational, trigonometric, and logarithmic.** Note: Functions may include linear, quadratic, exponential, polynomial, square root, cube root, piecewise defined (including step and absolute value), **rational, trigonometric, and logarithmic.**
	ELG.MA.HS.F.5 Build new functions from existing functions. o **F-BF.4** Find inverse functions. o **F-BF.4b (+)** Verify by composition that one function is the inverse of another. o **F-BF.4c (+)** Read values of an inverse function from a graph or a table, given that the function has an inverse. o **F-BF.4d (+)**Produce an invertible function from a non-invertible function by restricting the domain. o **F-BF.5 (+)**Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Students will demonstrate command of the ELG by:

- Graph/identify vertical stretches and vertical shrinks (dilations)
- Graph/identify a horizontal stretches and horizontal shrinks (dilations).
- Graph/identify vertical and horizontal translations.
- Graph/identify vertical and horizontal reflections.
- Identify/graph even and odd functions.
- Describe and explain the transformation of the parent function.
- Determine the inverse of a function.

Vocabulary:

- arithmetic sequence
- horizontal shrink/stretch (dilation)
- horizontal translation
- inverse function
- odd and even function
- parent function
- reflection
- transformation
- vertical shrink/stetch (dilation)
- vertical translation

Sample Instructional/Assessment Tasks:

1) **Standard(s): F-BF.3**
Source: https://www.illustrativemathematics.org/content-standards/HSF/BF/B/3/tasks/232

Item Prompt:

Determine whether each of these functions is odd, even, or neither. Use algebraic methods on all of the functions. You may start out by looking at a graph, if you need to.

1. $f(x) = 3^x + 3^{-x}$
2. $g(x) = 2^x - 2^{-x}$
3. $h(x) = x^2 + 4x - 2$
4. $j(x) = x^3 - 4x$

Correct Answer:

1. even
2. odd
3. neither even or odd
4. odd

2) **Standard(s): F-BF.4**

Source: https://www.illustrativemathematics.org/content-standards/HSF/BF/B/4/tasks/234

Item Prompt:

The table below shows the number of households in the U.S. in the years 1998-2004 [data source: www.census.gov].

households (in thousands)	97,107	98,990	99,627	101,018	102,528	103,874	104,705

a. Find a linear function, h, which models the number of households in the U.S. (in thousands) as a function of the year, t.

b. Write an expression for h^{-1}.

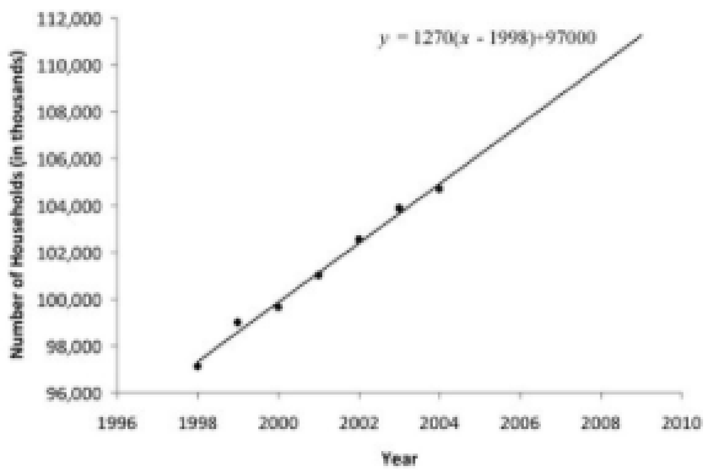
c. Find $h^{-1}(111,000)$ and interpret your answer in terms of the number of households.

Correct Answer:

a. The average rate of change in the number of households from 1998 to 2004 is

$$\frac{104,705 - 97,107}{2004 - 1998} \approx 1270 \text{ thousand/year.}$$

We choose this rate as the annual rate of change in our model, and we approximate the number of households in 1998 by $97,000$. This leads to the the linear function $h(t) = 1270(t - 1998) + 97,000$



b. The original function first subtracted 1998 from its input, then multiplied by 1270, and finally added 97,000. Thus, the inverse function would subtract 97,000 from its input, divide by 1270, and then add 1998. Thus,

$$h^{-1}(x) = \frac{(x - 97,000)}{1270} + 1998$$

c. Using the formula for the inverse function from b), we get $h^{-1}(111,000) \approx 2009$. This is the year the number of U.S. households reached 111,000 thousand. Of course, this is extrapolating well beyond the data, so the this prediction must be viewed with caution.