# ELG HS.F.6: **Construct and compare linear, quadratic, and exponential models and solve problems.**

## Vertical Progression:

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|---|---|
| **8th Grade** | **8.F.B Use functions to model relationships between quantities.**<br>o **8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. |
| **Algebra 1** | **ELG.MA.HS.F.6 Construct and compare linear, quadratic, and exponential models and solve problems.**<br>o **F-LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.<br>o **F-LE.1a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.<br>o **F-LE.1b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.<br>o **F-LE.1c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.<br>o **F-LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).<br>o **F-LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. |
| **Algebra 2** | **ELG.MA.HS.F.6 Construct and compare linear, quadratic, and exponential models and solve problems.**<br>o **F-LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).<br>o **F-LE.4** For exponential models, express as a logarithm the solution to $ab^{ct}=d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. |

## Students will demonstrate command of the ELG by:

- Constructing linear functions given arithmetic sequences.
- Constructing exponential functions given geometric sequences.
- Developing arithmetic and geometric sequences given multiple representations of data (table, graph, input-output pairs, or description).
- Applying linear and exponential functions to real-world applications.
- Applying arithmetic and geometric sequences to real-world applications.

## Vocabulary:

- arithmetic sequence
- exponential function
- geometric sequence
- linear function
- logarithm

## Sample Instructional/Assessment Tasks:

1) **Standard(s): F-LE**
**Source:** https://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/347

**Item Prompt:**

According to *Wikipedia*, the International Basketball Federation (FIBA) requires that a basketball rebound to a height of 1300 mm when dropped from a height of 1800 mm.

a. Suppose you drop a basketball and the ratio of each rebound height to the previous rebound height is 1300:1800. Let $h$ be the function that assigns to $n$ the rebound height of the ball (in mm) on the $n^{\text{th}}$ bounce or rebound. Complete the chart below, rounding to the nearest mm.

| $n$ | $h(n)$ |
|---|---|
| 0 | 1800 |
| 1 | |
| 2 | |
| 3 | |

b. Write an expression for $h(n)$.

c. Solve an equation to determine on which bounce the basketball will first have a rebound height of less than 100 mm.

**Correct Answer:**

a. **For n=1, 1300; for n=2, about 939; for n=3, about 678**
b. $\boldsymbol{h(n) = 1800\left(\frac{13}{18}\right)^n}$
c. **Ninth rebound**

2) **Standard(s): F-LE.4**

**Source:** https://www.illustrativemathematics.org/content-standards/HSF/LE/A/4/tasks/638

**Item Prompt:**

In 1966, a Miami boy smuggled three Giant African Land Snails into the country. His grandmother eventually released them into the garden, and in seven years there were approximately 18,000 of them. The snails are very destructive and had to be eradicated. According to the USDA, it took 10 years and cost $1 million to eradicate them.

a. Assuming the snail population grows exponentially, write an expression for the population, *P*, in terms of the number, *t*, of years since their release.
b. How long does it take for the population to double
c. Assuming the cost of eradicating the snails is proportional to the population, how much would it have cost to eradicate them if
   i. they had started the eradication program a year earlier?
   ii. they had let the population grow unchecked for another year?

**Correct Answers:**

a. $P(t) = 3e^{1.24t}$ for $0 \le t \le 7$
b. Just over half a year
c. i. about $280,000 ii. About $3,400,000