Vertical Progression:

Geometry	**ELG.MA.HS.G.7 Define trigonometric ratios and solve problems involving right triangles** o **G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. o **G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles. o **G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★
Algebra 2	**ELG.MA.HS.F.8 Extend the domain of trigonometric functions using the unit circle.** o **F-TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. o **F-TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
	ELG.MA.HS.F.8 Extend the domain of trigonometric functions using the unit circle. o **F-TF.3 (+)**Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. o **F-TF.4 (+)**Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Students will demonstrate command of the ELG by:

- Converting angle measures between radian measure and degree measure.
- Using the unit circle to extend trigonometric functions to all real numbers.

Vocabulary:

- counterclockwise
- radian measure
- trigonometric functions

Sample Instructional/Assessment Tasks:

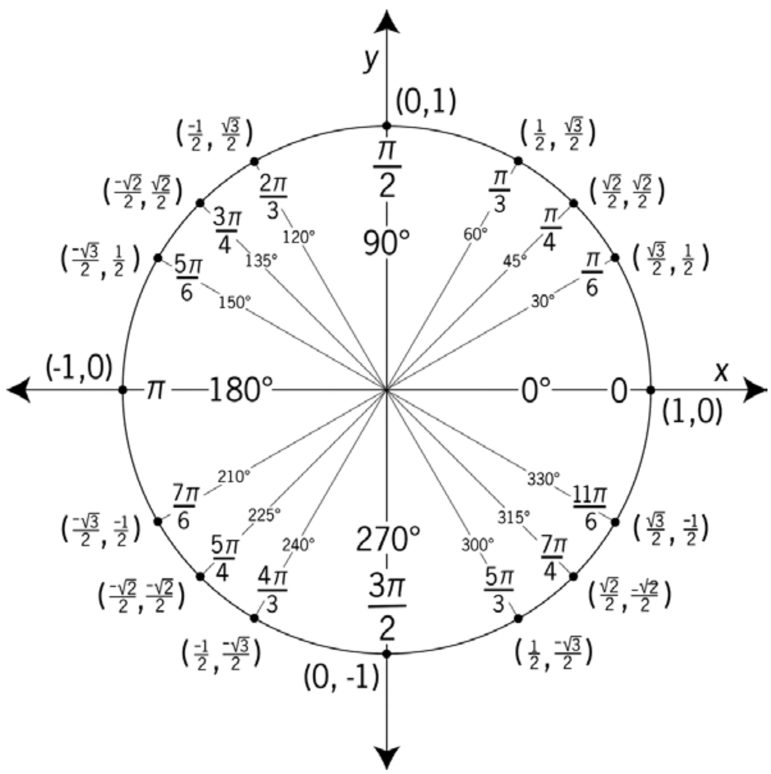
1) **Standard(s): F-TF.2**

Source: http://www.katm.org/flipbooks/HS%20FlipBook%20Final%20CCSS%202014.pdf

Item Prompt:

a. Using the Unit Circle below, find the following:

$\sin(-30°) =$	$\sin(330°) =$
$\tan(420°) =$	$\tan(60°) =$
$\cos(-90°) =$	$\cos(270°) =$
$\csc\left(\frac{\pi}{3}\right) =$	$\csc\left(\frac{13\pi}{3}\right) =$
$\cot(-45°) =$	$\cot(-765°) =$
$\sec\left(\frac{16\pi}{3}\right) =$	$\sec\left(\frac{4\pi}{3}\right) =$
$\csc\left(\frac{9\pi}{4}\right) =$	$\csc\left(\frac{\pi}{4}\right) =$
$\tan(4\pi) =$	$\tan(10\pi) =$
$\sin\left(\frac{3\pi}{2}\right) =$	$\sin\left(\frac{11\pi}{2}\right) =$
$\cos\left(\frac{-\pi}{6}\right) =$	$\cos\left(\frac{11\pi}{6}\right) =$



b. Look at each pair of angles above. What do you notice about those pairs?

c. What conclusions can you draw about the trigonometric functions and how they work about the circle?

Correct Answers:

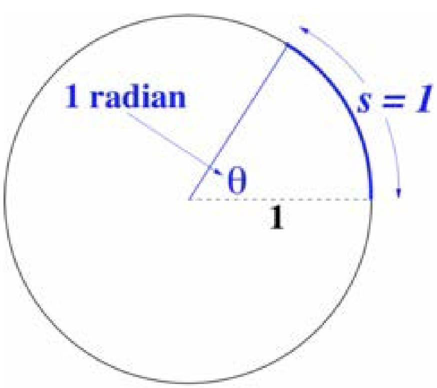
a. See grid below.
b. Both angles take you to the same point. The angle pairs are all co-terminal.
c. The functions repeat themselves as you go around and around the circle, forward/counter-clockwise or backward/clockwise.

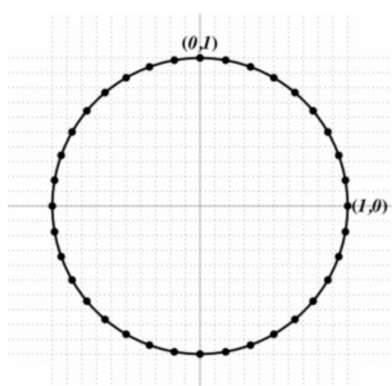
$\sin(-30°) = -\frac{1}{2}$	$\sin(330°) = -\frac{1}{2}$
$\tan(420°) = \sqrt{3}$	$\tan(60°) = \sqrt{3}$
$\cos(-90°) = 0$	$\cos(270°) = 0$
$\csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$	$\csc\left(\frac{13\pi}{3}\right) = \frac{2}{\sqrt{3}}$
$\cot(-45°) = -1$	$\cot(-765°) = -1$
$\sec\left(\frac{16\pi}{3}\right) = -2$	$\sec\left(\frac{4\pi}{3}\right) = -2$
$\csc\left(\frac{9\pi}{4}\right) = \frac{2}{\sqrt{2}}$	$\csc\left(\frac{\pi}{4}\right) = \frac{2}{\sqrt{2}}$
$\tan(4\pi) = 0$	$\tan(10\pi) = 0$
$\sin\left(\frac{3\pi}{2}\right) = -1$	$\sin\left(\frac{11\pi}{2}\right) = -1$
$\cos\left(\frac{-\pi}{6}\right) = \frac{\sqrt{3}}{2}$	$\cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$

2) **Standard(s): F-TF.1**
Source: https://www.illustrativemathematics.org/content-standards/HSF/TF/A/1/tasks/1874

Item Prompt:

Definition: An angle of 1 radian is defined to be the angle, in the counterclockwise direction, at the center of a unit circle which spans an arc of length 1. The picture below illustrates this definition.



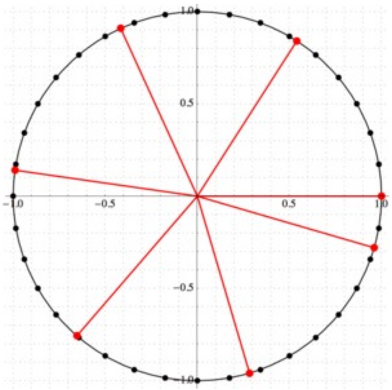


a. On the unit circle above, the angles 10° , 20° , 30° , etc., are indicated by black dots on the circle. Mark off angles of measure 0, 1, 2, 3, 4, 5, and 6 radians. Estimate the corresponding angle measure in degrees.

b. Estimate the angles in radians that correspond to 180° and 360°.

Correct Answer:

a. We can use a piece of string or a bendable ruler to measure the radius of the circle and trace the same distance around the circumference of the circle for 1, 2, 3, 4, 5, and 6 radians. The diagram below shows the corresponding angles.



We can see from the diagram that 180° corresponds to a little bit more than 3 radians. In fact, we can use the 10 degree points to help us get a better estimate. The arc subtended by 1 radian is divided into a little less than 6 equal pieces by the points and the arc subtended by 3 radians is about 1 piece short of half the circle, so 180° is about 3+16≈3.17 radians. Similarly, 360° is about 6.34 radians: these are both slight over estimates since the arc subtended by 3 radians is a little bit less than 1 piece short of 180°.