ELG HS.G.11: Translate between the geometric description and the equation for a conic section.

Vertical Progression:

8th Grade	**8.B.B Understand and apply the Pythagorean Theorem.** o **8.G.B.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Geometry	**ELG.MA.HS.G.11 Translate between the geometric description and the equation for a conic section.** o **G-GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
Algebra 2	**ELG.MA.HS.G.11 Translate between the geometric description and the equation for a conic section.** o **G-GPE.2** Derive the equation of a parabola given a focus and directrix.
	ELG.MA.HS.G.11 Translate between the geometric description and the equation for a conic section. o **G-GPE.3 (+)**Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Students will demonstrate command of the ELG by:

- Finding the distance from a point on the parabola (x, y) to the directrix.
- Finding the distance from a point on the parabola (x, y) to the focus using the distance formula (Pythagorean Theorem).
- Equating the two distance expressions for a parabola to write its equation.
- Identifying the focus and directrix of a parabola when given its equation.

Vocabulary:

- directrix
- equation of a parabola
- focus

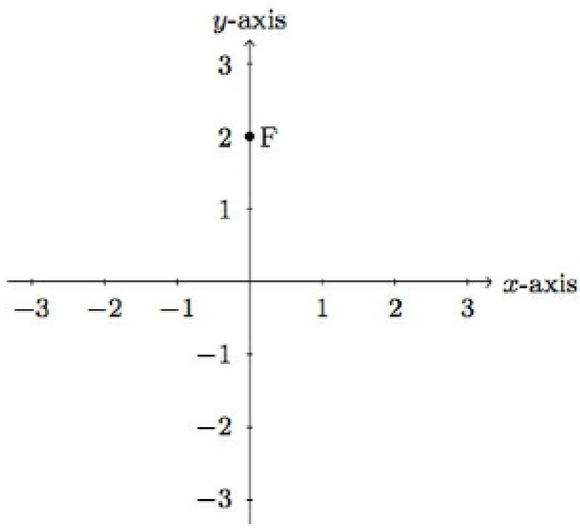
Sample Instructional/Assessment Tasks:

1) **Standard(s): G-GPE.2**

Source: https://www.illustrativemathematics.org/content-standards/HSG/GPE/A/2/tasks/1561

Item Prompt:

Suppose $F = (0, 2)$ and ℓ is the x-axis:



a. Find the point on the y-axis equidistant from F and ℓ. Label this point P_0.

b. Find the point on the line $x = 1$ which is equidistant from F and ℓ. Label this point P_1.

c. Find the point on the line $x = 2$ which is equidistant from F and ℓ. Label this point P_2.

d. If this process is repeated for the vertical lines $x = a$ for all real numbers a, what curve do the points P_a make?

Correct Answer:

a. The point equidistant from F and the y-axis is $P_0 = (0, 1)$, the bisector of the segment joining $(0, 0)$ and $(0, 2)$.

b. To find the point P_1 equidistant from F and the line $x = 1$, we know that $P_1 = (1, y)$ for some number y. The distance from $(1, y)$ to ℓ is y while its distance from F can be found using the Pythagorean Theorem:

$$\sqrt{1 + (y - 2)^2}.$$

So we need to solve the equation

$$y = \sqrt{1 + (y - 2)^2}.$$

Squaring both sides gives $y^2 = 1 + (y - 2)^2$ or $-4y + 5 = 0$. So $y = \frac{5}{4}$. This means that $P_1 = \left(1, \frac{5}{4}\right)$.

c. To find the point P_2 equidistant from F and the line $x = 2$, we know that $P_2 = (2, y)$ for some number y. The distance from $(2, y)$ to ℓ is y while its distance from F can be found using the Pythagorean Theorem:

$$\sqrt{4 + (y - 2)^2}.$$

So we need to solve the equation

$$y = \sqrt{4 + (y - 2)^2}.$$

Squaring both sides gives $y^2 = 4 + (y - 2)^2$ or $-4y + 8 = 0$. So $y = 2$ and $P_2 = (2, 2)$. In this case, we can readily see that P_2 is a distance of 2 from both F and ℓ.

d. We can try this procedure with $y = a$. The distance from (a, y) to ℓ is y while its distance from F is

$$\sqrt{a^2 + (y - 2)^2}$$

by the Pythagorean Theorem. Setting these two quantities equal and squaring both sides gives

$$y^2 = a^2 + (y - 2)^2.$$

After simplifying we find $-4y + a^2 + 4 = 0$ or $y = \frac{a^2+4}{4}$. This means that $P_a = \left(a, \frac{a^2+4}{4}\right)$. The set of all of these points is the graph of the function $y = \frac{x^2}{4} + 1$ which agrees with our calculations in parts a, b, and c above. The graph of this parabola, along with the plotted points, is shown here:

2) **Standard(s): G-GPE.2**

Source: Kansas Flip Book

Item Prompt:

Given the equation $20(y-5)=(x+3)^2$, find the focus, vertex and directrix.

Solution:

The vertex is at (-3, 5) and to find the vertex we know that the constant of the unsquared term is 20. Since 4*p* = 20 then *p* = 5. The focus is 5 units above the vertex at (-3, 5+5) or (-3, 10). The directrix is 5 units below the vertex, so *y* = 0.