Algebra 2

ELG HS.S.5: Make inferences and justify conclusions from sample surveys, experiments, and observational studies

Vertical Progression:

7th Grade	**7.SP.A Use random sampling to draw inferences about a population.** o **7.SP.A.1** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. o **7.SP.A.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.
Algebra 2	**ELG.MA.HS.S.5 Make inferences and justify conclusions from sample surveys, experiments, and observational studies** o **S-IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. o **S-IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. o **S-IC.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. o **S-IC.6** Evaluate reports based on data.

Students will demonstrate command of the ELG by:

- Identifying situations as sample survey, experiment, or observational study, discussing the appropriateness of each one's use in contexts with limiting factors, and describing the purposes and differences of each.
- Designing or evaluating sample surveys, experiments and observational studies with randomization and discussing the importance of randomization in these processes.
- Explaining techniques/applications for randomly selecting study subjects from a population and how those techniques/applications differ from those used to randomly assign existing subjects to control groups or experimental groups in a statistical experiment.
- Calculating a sample mean and proportion and using these to estimate population values.
- Calculating the sample mean and standard deviation of two treatment groups and the difference of the means, conducting a simulation for each treatment group using the sample results as the parameters for the distributions, calculating the difference of means for each simulation, and representing those differences in a histogram.
- Using the results of the simulation to create a confidence interval for the difference of means.
- Using the confidence interval to determine if the parameters are significantly different based on the original difference of means.
- Reading and explain in context data from outside reports, identifying the variables as quantitative or categorical, describing how the data were collected, indicating any potential biases or flaws and identifying inferences the author of the report made from sample data.
- Writing or presenting a summary of a data-based report addressing the sampling techniques used, inferences made, and any flaws or biases.

Vocabulary:

- bias
- confidence interval
- control group
- experiment
- experimental group
- observational study
- population
- randomization
- sample mean
- sample standard deviation
- sample survey
- simulation

Sample Instructional/Assessment Tasks:

1) **Standard(s): S-ID.3**

Source: PARCC Algebra 2 PBA Practice Test

Item Prompt:
For a statistics project, a group of students decide to collect data in order to approximate the percent of people in the town who are left-handed. They ask every third student entering the school cafeteria whether he or she is left-handed or right-handed. What type of method did this group use? Explain which population the group can draw a conclusion about based on their method. Suggest a better method that would allow the students to draw a conclusion about all the residents in their town.

Correct Answer:

The students used a random sample survey method. The group can draw a valid conclusion about the population of students eating in the school cafeteria because their sample only includes students entering the school cafeteria. The group can still use a sample survey, but they can use a random sampling from the whole town rather than limiting the sample to the school cafeteria.

2) **Standard(s): S-ID.4**

Source: https://www.illustrativemathematics.org/content-standards/HSS/IC/B/4/tasks/1956

Item Prompt:

Researchers have questioned whether the traditional value of 98.6°F is correct for a typical body temperature for healthy adults. Suppose that you plan to estimate mean body temperature by recording the temperatures of the people in a random sample of 10 healthy adults and calculating the sample mean. How accurate can you expect that estimate to be? In this activity, you will develop a margin of error that will help you to answer this question.

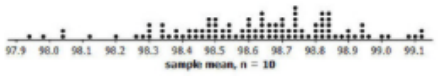
Let's assume for now that body temperature for healthy adults follows a normal distribution with mean 98.6 degrees and standard deviation 0.7 degrees. Here are the body temperatures for one random sample of 10 healthy adults from this population:

97.73	98.76	98.27	99.95	98.47	98.49	98.97	98.68	99.27	99.25

a. What is the mean temperature for this sample?

b. If you were to take a different random sample of size 10, would you expect to get the same value for the sample mean? Explain.

Below is a dot plot of the sample mean body temperature for 100 different random samples of size 10 from a population where the mean temperature is 98.6 degrees.



c. How many of the samples had sample means that were greater than 98.5 degrees and less than 98.7 degrees?

d. Based on the dot plot above, if you were to take a different random sample from the population, would you be surprised if you got a sample mean of 98.8 or greater? Explain why or why not.

e. Which of the following statements is appropriate based on the dot plot of sample means above?

Statement 1:Most random samples of size 10 from the population would result in a sample mean that is within 0.1 degrees of the value of the population mean (98.6).

Statement 2:Most random samples of size 10 from the population would result in a sample mean that is within 0.3 degrees of the value of the population mean (98.6).

Statement 3:Most random samples of size 10 from the population would result in a sample mean that is within 0.5 degrees of the value of the population mean (98.6).

The **margin of error** associated with an estimate of a population mean can be interpreted as the maximum likely difference between the estimate and the actual value of the population mean for a given sample size.

f. Explain why 0.45 degrees would be a reasonable estimate of the margin of error when using the sample mean from a random sample of size 10 to estimate the mean body temperature for the population described above.

g. If you were to use a random sample of size 20 to estimate the population mean, would you expect the estimate to be closer to or farther from the actual value of the population mean than if you had used a random sample of size 10? Would this mean that the margin of error would be less than or greater than the margin of error for a sample of size 10?

Correct Answer:

a. 98.78 degrees

b. No. The value of the sample mean will vary from sample to sample because different samples will include different individuals from the population.

c. 25

d. No. Many of the 100 random samples resulted in a sample mean that was 98.8 or greater. This would not be surprising.

e. Statement 3 (although some students may say Statement 2, depending on how they interpret "most").

f. Only 9 of the 100 samples had sample means that were farther from 98.6 degrees than 0.45 degrees. This means that 91 of the 100 random samples resulted in a sample mean that was closer to the actual value of the population mean than 0.45 degrees. It would be unlikely to get a random sample of size 10 that had a sample mean farther away from the population mean than 0.45 degrees.

g. Would expect it to be closer because the sample size of 20 is larger than the sample size 10. This would mean that we expect the margin of error to be smaller.