ELG HS.S.6: Understand independence and conditional probability and use them to interpret data.

Vertical Progression:

7th Grade	**7.SP.C Investigate chance processes and develop, use and evaluate probability models.** o **7.SP.C.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. o **7.SP.C.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. o **7.SP.C.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. o **7.SP.C.7a** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. o **7.SP.C.7b** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.
Algebra 2	**ELG.MA.HS.S.6 Understand independence and conditional probability and use them to interpret data.** o **S-CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). o **S-CP.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. o **S-CP.3** Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B),* and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. o **S-CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results*. o **S-CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer*.

Students will demonstrate command of the ELG by:

- Defining a sample space and events within the sample space.
- Establishing events as subsets of a sample space and using correct set notation, with appropriate symbols, to identify sets and subsets.
- Drawing Venn diagrams that show relationships between sets within a sample space.
- Defining and identifying independent events.
- Explaining properties of Independence and Conditional Probabilities.
- Calculating probabilities for events, including joint probabilities, using various methods (e.g., Venn diagrams, frequency table).
- Predicting if two events are independent, explaining reasoning and checking.
- Defining dependent events and conditional probability.
- Determining if two events are independent and justifying the conclusion.

Vocabulary:

- sample space
- Venn diagram

Sample Instructional/Assessment Tasks:

1) **Standard(s): S-CP.1,4, 6**

Source: https://www.illustrativemathematics.org/content-standards/tasks/949

Item Prompt:

On April 15, 1912, the Titanic struck an iceberg and rapidly sank with only 710 of her 2,204 passengers and crew surviving. Data on survival of passengers are summarized in the table below. (Data source: http://www.encyclopedia-titanica.org/titanic-statistics.html)

	Survived	Did not survive	Total
First class passengers	201	123	324
Second class passengers	118	166	284
Third class passengers	181	528	709
Total passengers	500	817	1317

a. Calculate the following probabilities. Round your answers to three decimal places.

i. If one of the passengers is randomly selected, what is the probability that this passenger was in first class?

ii. If one of the passengers is randomly selected, what is the probability that this passenger survived?

iii. If one of the passengers is randomly selected, what is the probability that this passenger was in first class and survived?

iv. If one of the passengers is randomly selected from the first class passengers, what is the probability that this passenger survived? (That is, what is the probability that the passenger survived, given that this passenger was in first class?)

v. If one of the passengers who survived is randomly selected, what is the probability that this passenger was in first class?

vi. If one of the passengers who survived is randomly selected, what is the probability that this passenger was in third class?

b. Why is the answer to part (a.iv) larger than the answer to part (a.iii)?

c. Why is the answer to part (a.v) larger than the answer to part (a.vi)?

Correct Answer:

a. i. The probability of the passenger being in first class is the number of all first class passengers divided by total number of passengers , that is $P(\text{passenger being in first class}) = \frac{324}{1317} \approx 0.246$

ii. The probability that the passenger survived is the number of all passengers who survived divided by total number of passengers, that is $P(\text{passenger survived}) = \frac{500}{1317} \approx 0.380$

iii. This is the fraction of all passengers that are both in first class and survived, which is
$P[(\text{passenger was in first class}) \text{ and } (\text{passenger survived})] = \frac{201}{1317} \approx 0.153$

iv. This is a conditional probability. To find the probability that the passenger survived, given this passenger was in first class, we calculate the fraction of first class passenger who survived, that is
$P(\text{passenger survived} | \text{passenger was in first class}) = \frac{201}{324} \approx 0.620$

v. This is a conditional probability:
$P(\text{passenger was in first class} | \text{passenger survived})$ We can calculate it as the fraction of surviving passengers who were in first class, which is $\frac{201}{500} \approx 0.402$

vi. This is a conditional probability:
$P(\text{passenger was in third class} | \text{passenger survived})$ We can calculate it as the fraction of surviving passengers who were in third class, which is $\frac{181}{500} \approx 0.362$

b. Even though in both parts (a.iii) and (a.iv) we have the same numerator (201), in part (a.iii) the sample space consists of all the passengers, but in part (a.iv) the sample space is restricted to only the first class passengers. Since in part (a.iv) we divide by a smaller number, the answer in part (a.iv) is larger than in part (a.iii).

c. In both parts (a.v) and (a.vi) the sample space is restricted to all the passengers who survived. But since among that group there were more first class than third class passengers, the answer to part (a.v) is larger than the answer to part (a.vi).

2) **Standard(s): SP-CP.4-6**

Source: http://www.insidemathematics.org/assets/problems-of-the-month/friends%20you%20can%20count%20on.pdf

Item Prompt:

A survey was conducted by the local newspaper in your community. The survey sampled students in your school about the use of drugs. It stated that through the anonymous survey that 12% of the students indicated that they experimented with or currently use drugs. The survey has alarmed the community. Parents and community members are very concerned. The school board has been discussing the issue of drugs. They want to take strong action.

You have just learned that the school board is considering requiring a drug test for all the 1,200 students who attend your school. A test would be given twice a year. If a student failed the drug test, then the student would be expelled from school.

Most of the students are upset and nervous about such a test. They are saying, “How do we know the tests are accurate?” “What if you are taking medication for some ailment, will that indicate that you are taking illegal drugs?” “What happens if you get a false positive reading?” “How long after you take a drug will the test show positive?” “What if you stopped taking a prescribed drug for more than three months, will you still test positive?”

You want to be a friend to your school and classmates. You know that this drug test will cause a crisis at your school. The school board feels a lot of pressure to take action. You want to stop the school board from voting for this drug test. You know it will take a convincing argument to change their minds. If you are not careful in your presentation, it may look like you are defending drug use. That is the last thing that you need to have happen. You must find a way to defend the innocent and show that some students may get hurt by the test.

You decide to research the test. You call the drug testing company that the school board is considering hiring and ask for documentation on their tests. In their literature, it states that the tests are accurate 96% of the times they are administered.

You start to consider the information you have available. If the newspaper survey was accurate that 12% take drugs, how many of the students at your school supposedly take drugs? How many students are drug-free? If all the students are required to take the drug test, how many of the students’ tests will be accurate? How many of the tests will be inaccurate? How many students who do not take drugs will have a test that wrongly shows that they do take drugs? How many of the students who use drugs (either experimentally or regularly) will have an accurate test?

You are getting ready to present your argument to the board. Write an open letter to the board using mathematics to argue against general drug testing for all students.

Correct Answer:
A detailed explanation of the problem along with samples of student work can be found via the link above.