

### Vertical Progression:

<p><b>8<sup>th</sup> Grade</b></p>	<p><b>8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.</b></p> <ul style="list-style-type: none"> <li>○ <b>8.G.A.1</b> Verify experimentally the properties of rotations, reflections, and translations: <ul style="list-style-type: none"> <li>○ <b>8.G.A.1.a</b> Lines are taken to lines, and line segments to line segments of the same length.</li> <li>○ <b>8.G.A.1.b</b> Angles are taken to angles of the same measure.</li> <li>○ <b>8.G.A.1.c</b> Parallel lines are taken to parallel lines.</li> </ul> </li> <li>○ <b>8.G.A.2</b> Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.</li> <li>○ <b>8.G.A.3</b> Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</li> <li>○ <b>8.G.A.4</b> Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</li> </ul>
<p><b>Algebra 1</b></p>	<p><b>ELG.MA.HS.F.5 Build new functions from existing functions.</b></p> <ul style="list-style-type: none"> <li>○ <b>F-BF.3</b> Identify effects on graphs of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find values of <math>k</math> given the graphs. Experiment with cases and illustrate explanations of effects on graphs using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</li> </ul> <p>Note: Functions may include linear, quadratic, exponential, polynomial (quadratic or cubic), square root, cube root, and piecewise-defined functions (including step and absolute value).</p>
<p><b>Geometry</b></p>	<p><b>ELG.MA.HS.G.1 Experiment with transformations in the plane</b></p> <ul style="list-style-type: none"> <li>○ <b>G-CO.1</b> Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.</li> <li>○ <b>G-CO.2</b> Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).</li> <li>○ <b>G-CO.3</b> Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.</li> <li>○ <b>G-CO.4</b> Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.</li> <li>○ <b>G-CO.5</b> Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.</li> </ul>
<p><b>Algebra 2</b></p>	<p><b>ELG.MA.HS.F.5 Build new functions from existing functions.</b></p> <ul style="list-style-type: none"> <li>○ <b>F-BF.3</b> Identify effects on graphs of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find values of <math>k</math> given the graphs. Experiment with cases and illustrate explanations of effects on graphs using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</li> </ul> <p>Note: Functions may include linear, quadratic, exponential, polynomial, square root, cube root, piecewise defined (including step and absolute value), <b>rational, trigonometric, and logarithmic.</b></p>

#### Students will demonstrate command of the ELG by:

- Demonstrating their knowledge of precise definitions of angles, line, point, plane, circles, perpendicular and parallel lines, and line segments.
- Using proper notation.
- Representing transformations in the plane using tools.
- Describing transformations as functions.
- Comparing transformations that preserve distance and angle to those that do not.
- Describing the rotations and reflections that carry a given polygon onto itself.
- Developing definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- Drawing the transformed figure of a geometric figure using tools.
- Specifying a sequence of transformations that will carry a given figure onto another.

#### Vocabulary:

- angle
- circular arc
- dilations
- distance
- isometry
- line
- line segment
- parallel lines
- perpendicular lines
- plane
- point
- reflections
- rotations
- transformation
- translations

#### Sample Instructional/Assessment Tasks:

##### 1) Standard(s): G-CO.A..1

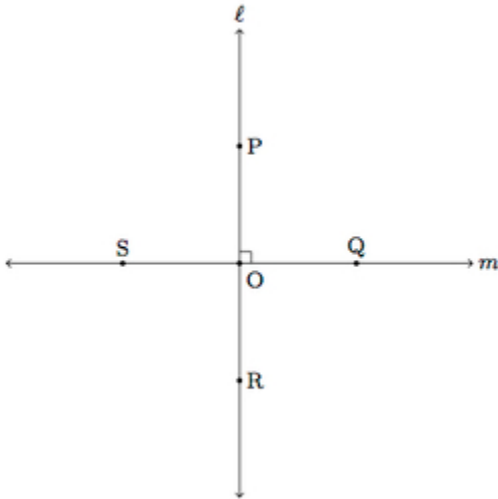
Source: <https://www.illustrativemathematics.org/content-standards/HSG/CO/A/1/tasks/1544>

**Item Prompt:** Three students have proposed these ways to describe when two lines  $\ell$  and  $m$  are perpendicular:

- $\ell$  and  $m$  are perpendicular if they meet at one point and one of the angles at their point of intersection is a right angle.
  - $\ell$  and  $m$  are perpendicular if they meet at one point and all four of the angles at their point of intersection are right angles.
  - $\ell$  and  $m$  are perpendicular if they meet at one point and reflection about  $\ell$  maps  $m$  to itself.
- Explain why each of these definitions is correct. What are some of the advantages and disadvantages with each?

**Correct Answer:**

- a. Below is a picture of this situation which we will use for (a) and then (b). Here we have assumed that  $\angle POQ$  is a right angle.



The main advantage to this definition is its simplicity. It only makes an assumption about one of the four angles at O, as opposed to Definition (b) which assumes that all four are right angles. Also, unlike Definition (c) the only prior knowledge assumed is the definition of a right angle.

We also show here that Definition (a) implies Definition (b) (with fewer hypotheses). Note that angles POQ and QOR are supplementary since together they form  $\ell$ . Since  $m(\angle POQ)=90$  by hypothesis and  $m(\angle POQ)+m(\angle QOR)=180$  this means that  $m(\angle QOR)=90$ , so it is also a right angle. The same reasoning can be applied to conclude that  $m(\angle POS)=90$  since, together with  $\angle POQ$ , this makes line  $m$ . Finally, for  $\angle SOR$ , it makes a vertical pair together with right angle  $\angle POQ$  and so it is congruent to  $\angle POQ$ . Therefore  $m(\angle SOR)=90$ . Thus all four angles made by  $\ell$  and  $m$  at O are right angles.

- b. This definition has the same prerequisites as (a), namely understanding of right angles. Unlike Definition (a), it assumes that all four angles made by  $\ell$  and  $m$  are right angles. Though this can be deduced as in the last paragraph above, this definition has the advantage of being natural: no one of the four angles is given special status as in the first definition. One disadvantage to this definition is that if we wish to show or deduce that a pair of lines  $\ell$  and  $m$  are perpendicular, we have four different angle computations to make instead of only one with the first definition.
- c. This definition is accurate but is problematic because the usual definition of reflection about  $\ell$  for a point A not on  $\ell$  is the following: reflection about  $\ell$  maps A to B where  $\ell$  is the perpendicular bisector of segment AB. Thus the definition of reflection about  $\ell$  assumes that we know what it means for line  $\ell$  to be perpendicular to segment AB. But this is the same as what it means for  $\ell$  and line AB to be perpendicular. In other words, reflections presume knowledge of what it means for two lines to be perpendicular. Therefore, reflections should not be used in the definition of perpendicular lines. It is important, however, for students to associate perpendicular lines to the concept of reflections even if reflections are not appropriate to define perpendicular lines.

### 2) Standard(s): G-CO.A.3

Source: <https://www.illustrativemathematics.org/content-standards/HSG/CO/A/3/tasks/1471>

#### Item Prompt:

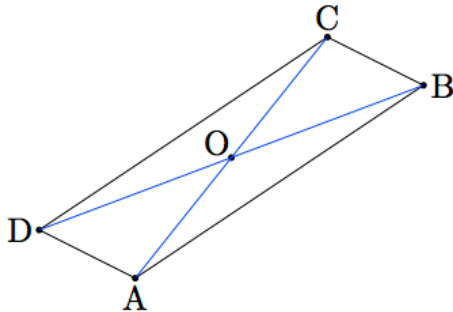
Suppose ABCD is a quadrilateral for which there is exactly one rotation, through an angle larger than 0 degrees and less than 360 degrees, which maps it to itself. Further, no reflections map ABCD to itself. What shape is ABCD?

#### Solution:

Suppose  $r$  is the rotation which maps ABCD to itself. Applying  $r$  twice will then also carry ABCD to itself. Since there was only one non-identity rotation that mapped ABCD to itself, it must be the case that applying  $r$  twice is the identity. This means that  $r$  rotates through exactly half of a full circle, or 180 degrees.

Next we examine the effect of  $r$  on the sides of the quadrilateral. A rotation by 180 degrees takes each line  $\ell$  to a line  $r(\ell)$  which is parallel to  $\ell$ . Suppose  $O$  is the center of the rotation. This point could lie on zero, one, or two of the lines containing the sides of ABCD: in all cases, there is at least one pair of these lines, through a vertex of ABCD, neither of which contains  $O$ . The 180 degree rotation takes each of these lines to a distinct parallel line. This gives two different pairs of parallel lines containing opposite sides of ABCD (and also shows that  $O$  does not lie on any of the four lines containing the sides of ABCD). So ABCD is a parallelogram.

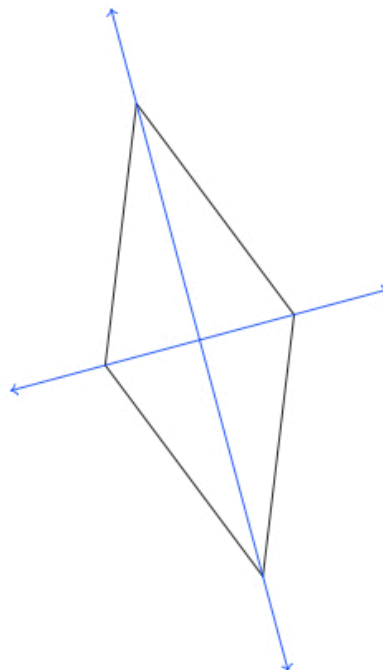
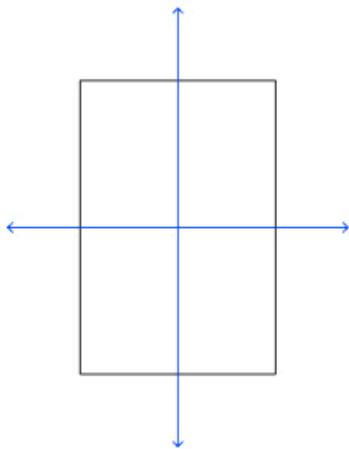
Before proceeding to examine ABCD further, we provide a picture:



A 180 degree rotation interchanges A and C and also interchanges B and D. Moreover, O is the midpoint of both of segments AC and BD. This particular parallelogram has no other symmetries, save for the identity. Certain special parallelograms do admit other symmetries: for example a rectangle has two lines of reflective symmetry in addition to a 180 degree rotation (plus additional symmetries if the rectangle happens to be a square). Another special class of parallelograms which admit additional symmetries are the rhombuses: these also have two lines of reflective symmetry (passing through opposite vertices) like a rectangle. So we conclude with a final answer of:

***Quadrilateral ABCD is a parallelogram which is neither a rectangle nor a rhombus.***

Below are pictures of a rectangle and a rhombus with their lines of symmetry:



Note that a square is both a rectangle and a rhombus and has 4 lines of symmetry as a result.