ELG HS.G.12: **Use coordinates to prove simple geometric theorems algebraically.**

Vertical Progression:

7th Grade	**7.RP.A Analyze proportional relationships and use them to solve real-world and mathematical problems.** o **7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units, areas, and other quantities measured in like or different units. **7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** o **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
8th Grade	**8.EE.B Understand the connections between proportional relationships, lines, and linear equations.** o **8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of the two moving objects has greater speed.* **8.B.B Understand and apply the Pythagorean Theorem.** o **8.G.B.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Geometry	**ELG.MA.HS.G.12 Use coordinates to prove simple geometric theorems algebraically.** o **G-GPE.4** Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2). o **G-GPE.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). o **G-GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. o **G-GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Students will demonstrate command of the ELG by:

- Calculating the distance, midpoint and slope given the coordinates of two points.
- Showing parallel lines have the same slope and the same rate of change.
- Showing perpendicular lines have opposite reciprocal slopes and intersect at a right angle.
- Determining the coordinates of a point of a given partition on a directed segment.
- Choosing the appropriate formula and calculating the perimeter or area of a given polygon using the coordinates of the polygon.
- Using coordinates to prove simple geometric theorems algebraically.

Vocabulary:

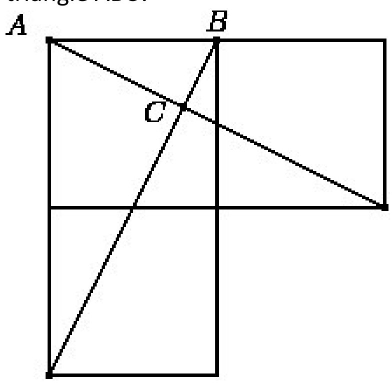
- coordinate proof
- directed line segment
- distance formula
- logical reasoning
- midpoint formula
- opposite recriprocal
- parallel lines
- perpendicular lines
- point-slope
- polygons
- Pythagorean theorem
- rate of change
- ratio
- slope formula
- slope-intercept

Sample Instructional/Assessment Tasks:

1) **Standard(s): G-GPE.B.5, G-GPE.B.4, G-CO.A, G-SRT.B.5**
Source: https://www.illustrativemathematics.org/content-standards/HSG/GPE/B/4/tasks/918

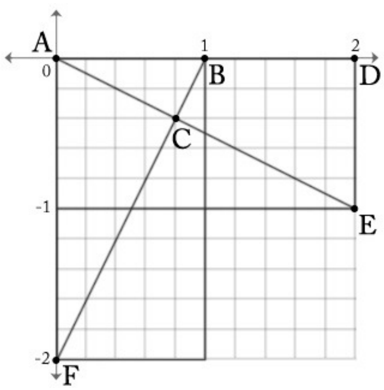
Item Prompt:

Three unit squares and two line segments connecting two pairs of vertices are shown. What is the area of triangle ABC?



Correct Answer:

We begin by placing the three unit squares on an x-y coordinate system so that A is the origin and B=(1,0):



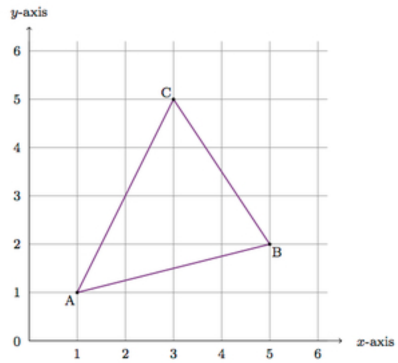
Points D, E, and F are labelled in the picture. A linear equation whose solutions are the line $\overleftrightarrow{AE}$ is $y = -\frac{1}{2}x$ since A = (0,0) and E = (2,−1) are both solutions to this linear equation. A linear equation whose solutions are the line $\overleftrightarrow{BF}$ is $y = 2x - 2$ since B = (1,0) and F = (0,−2) are both solutions to this equation. The point C lies on both of these lines and so the coordinates of C simultaneously solve the two equations. If we solve these equations we find C = $(\frac{4}{5}, -\frac{2}{5})$. Using $\overline{AB}$ as a base, the altitude of the triangle is $\frac{2}{5}$ since C is $\frac{2}{5}$ below the x-axis. So the area of △ABC is $\frac{1}{2} \cdot 1 \cdot \frac{2}{5} = \frac{1}{5}$

2) **Standard(s) : ELG.MA.HS.G.12 (HSG-GPE.B.6)**

Source: https://www.illustrativemathematics.org/content-standards/HSG/GPE/B/6/tasks/1867

Item Prompt:

Below is a picture of △ABC with vertices lying on grid points:



a. Draw the image of △ABC when it is scaled with a scale factor of $\frac{1}{3}$ about the vertex A
   1. Label this triangle A′B′C′ and find the coordinates of the points A′, B′, and C′.
b. Draw the image of △ABC when it is scaled with a scale factor of $\frac{2}{3}$ about the vertex B
   1. Label this triangle A′′B′′C′′ and find the coordinates of the points A′′, B′′, and C′′.
c. How does A′′ compare B′? Why?

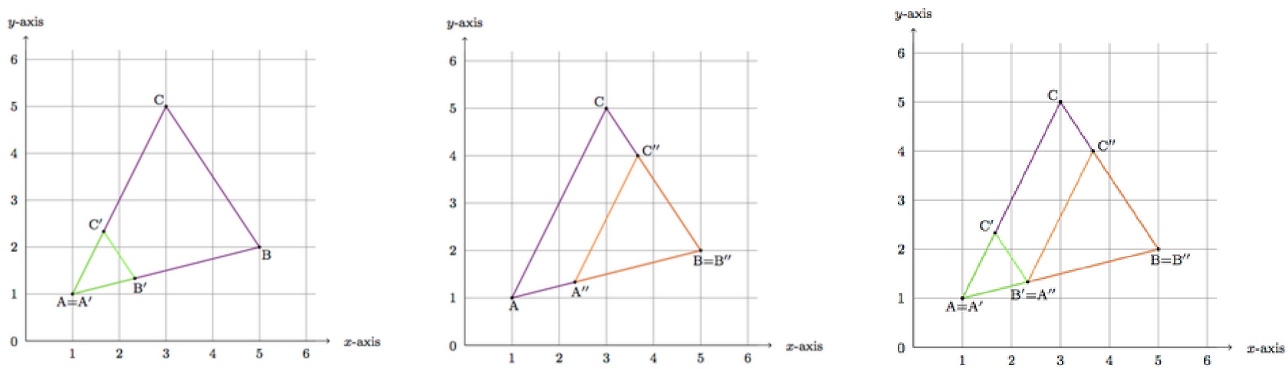
Solution:

a. Since A is the center of the dilation, it does not move and we will have A′=A. For vertex B, we know that B′ will lie on $\overleftrightarrow{AB}$ because dilations preserve lines through the center of dilation. Since the scale factor is $\frac{1}{3}$ we know that $|AB'| = \frac{1}{3}|AB|$. From A to B is 4 units to the right and one unit up. Therefore, one third of the way from A to B (along $\overline{AB}$) will be the point $B' = \left(1 + \frac{4}{3}, 1 + \frac{1}{3}\right)$. Applying this technique to C which is 2 units to the right and 4 units up from A, we find $C' = \left(1\frac{2}{3}, 2\frac{1}{3}\right)$. The scaled triangle A′B′C′ is pictured below:

b. Since B is the center of the dilation, it does not move and we will have B′′=B. For vertex A, we know that A′′ will lie on $\overleftrightarrow{AB}$ because dilations preserve lines through the center of dilation. Since the scale factor is $\frac{2}{3}$ we know that $|BA''| = \frac{2}{3}|BA|$. From B to A is 4 units to the left and one unit down. Therefore, two thirds of the way from B to A (along $\overline{AB}$) will be the point $A'' = \left(4 - \frac{8}{3}, 2 - \frac{2}{3}\right)$. Applying this technique to C which is 2 units to the left and 3 units up from B, we find $C'' = \left(3\frac{2}{3}, 4\right)$.

The scaled triangle A′′B′′C′′ is pictured below:

c. The calculations above show that $B' = A'' = \left(2\frac{1}{3}, 1\frac{1}{3}\right)$. To see why these vertices are the same, note that the sum of the dilation factors $\frac{1}{3}$ and $\frac{2}{3}$ is 1. The dilation about A maps $\overline{AB}$ to a the first third of $\overline{AB}$ while the dilation about B maps $\overline{AB}$ to the second two thirds of this segment. The two scaled triangles with a shared vertex are pictured below:



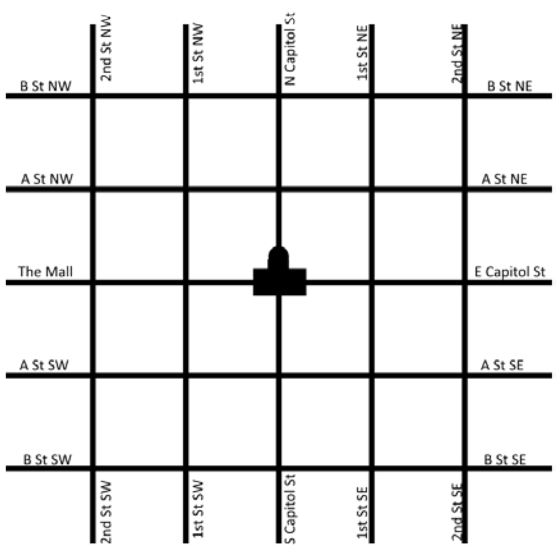
3) Standard(s): G-GPE.6

Source: Jonathan Mattes-Ritz

Item Prompt:

Pam is training carrier pigeons in Washington, DC. She lives in the Howard neighborhood at the intersection of 9th St and V St NW. She's trying to send a message to her friend Jon, who lives near Capitol Hill at the intersection of 13th St and D St SE. Unfortunately, Pam's pigeons don't have much endurance. They usually give up $\frac{2}{7}$ of the way to their destination. Where should Pam go to pick up her tired pigeons?

Note: Washington DC is organized on a grid system with number streets running North/South and lettered streets running East/West. The US Capitol is at the center of the grid with each of the four quadrants labeled by direction (NE, NW, SW, SE). The numbers and letters go up as you move away from the Capitol.



Correct Answer:

Convert the address intersections to coordinate pairs.

Pam => 9th and V NW => (-9 , 22)

Jon => 13th and D SE => (13 , -4)

Find $\frac{2}{7}$ of the horizontal and vertical distances.

Horizontal: $\frac{2}{7}\big((13)-(-9)\big) \approx 6.3$

Vertical: $\frac{2}{5}\big((-4)-(22)\big) \approx -7.4$

Add those distances to Pam's coordinates.

Pigeon = $\big((-9)+(6.3)\,,\ (22)+(-7.4)\big) = (-7.7\,, 14.6)$

Convert the coordinates to the nearest intersection

$(-7.7\,, 14.6) \approx$ 8th St and N St NW.

Pam should go to the intersection of 8th St and N St NW to look for her bird.