Vertical Progression:

7th Grade	**7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** o **7.G.B.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. o **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
8th Grade	**8.G.C Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.** o **8.G.C.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
Algebra 1	**ELG.MA.HS.A.7 Create equations that describe numbers or relationships.** o **A-CED.A.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. **ELG.MA.HS.N.3 Reason quantitatively and use units to solve problems.** o **N-Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. o **N-Q.A.2** Define appropriate quantities for the purpose of descriptive modeling.
Geometry	**ELG.MA.HS.G.13 Explain volume formulas and use them to solve problems.** o **G-GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* o **G-GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Students will demonstrate command of the ELG by:

- Giving informal arguments for the formulas for circumference and area of a circle and volume of cylinders, pyramids, and cones.
- Using surface area and volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Vocabulary:

- Cavalieri's principle
- circumference
- cone
- cylinder
- dissection argument
- informal limit argument
- pyramid
- sphere
- volume

Sample Instructional/Assessment Tasks:

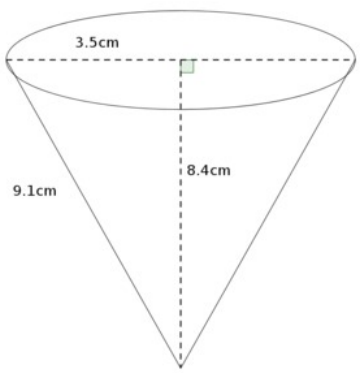
1) **Standard(s): G-GMD.A.3**
Source: https://www.illustrativemathematics.org/content-standards/tasks/527

Item Prompt:

Jared is scheduled for some tests at his doctor's office tomorrow. His doctor has instructed him to drink 3 liters of water today to clear out his system before the tests. Jared forgot to bring his water bottle to work and was left in the unfortunate position of having to use the annoying paper cone cups that are provided by the water dispenser at his workplace. He measures one of these cones and finds it to have a diameter of 7cm and a slant height (measured from the bottom vertex of the cup to any point on the opening) of 9.1cm.
Note: 1 cm^3=1 ml

a. How many of these cones of water must Jared drink if he typically fills the cone to within 1cm of the top and he wants to complete his drinking during the work day?
b. Suppose that Jared drinks 25 cones of water during the day. When he gets home he measures one of his cylindrical drinking glasses and finds it to have a diameter of 7cm and a height of 15cm. If he typically fills his glasses to 2cm from the top, about how many glasses of water must he drink before going to bed?

Solution:



a. Each cone of water contains approximately 13πr2h=13π(3.08)2(7.4)≈73.51 cubic centimeters (i.e., milliliters) of water.

b. He needs to drink just over 2 glasses of water before going to bed.

2) **Standard(s): G-GMD.A.1**

Source: https://www.illustrativemathematics.org/content-standards/HSG/GMD/A/1/tasks/1920

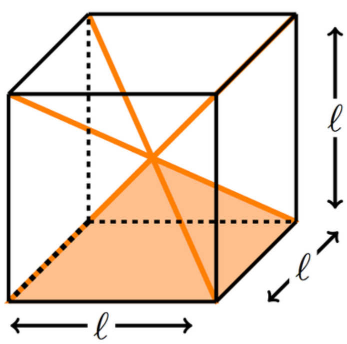
Item Prompt:

The four diagonals of a cube with side length ℓ meet in a point P, and divide the cube into six rectangular pyramids with square bases.

a. What is the height of each of these pyramids?
b. What is the volume of each of these pyramids?
c. It seems reasonable to suppose that the volume of a rectangular pyramid is, like a rectangular prism, proportional to its length ℓ, width w, and height h. That is, we expect a formula of the form V=c(ℓwh) for some constant c. Find a constant c for which this formula is true for the square pyramid described in the previous parts.
d. Does the dissection method described in this problem work to find a formula for the volume of an arbitrary pyramid? Why?

Solution:

Here is a picture of the scenario. Note that the congruence of the 6 pyramids can be viewed either dynamically (rotating the cube takes one pyramid onto another) or by calculating that all corresponding lengths are equal.



a. The height of the pyramid is ℓ/2.
b. The volume of each pyramid is $16\ell^3$.
c. C=13
d. The method used here is very special to this particular shape of pyramid. First it is vital that the base of the pyramid is a square. Secondly it is vital that the height of the pyramid be half the length of the base sides so that the six pyramids fit together perfectly to make a cube. If the height were smaller or larger they would no longer fit together to make a cube and so some other argument would be needed to calculate the volume.