ELG HS.G.14: **Visualize relationships between two-dimensional and three-dimensional objects.**

Vertical Progression:

7th Grade	**7.G.A Draw, construct, and describe geometrical figures and describe the relationships between them.** o **7.G.A.3** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
8th Grade	**8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.** o **8.G.A.1** Verify experimentally the properties of rotations, reflections, and translations. o **8.G.A.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. o **8.G.A.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
Geometry	**ELG.MA.HS.G.14 Visualize relationships between two-dimensional and three-dimensional objects.** o **G-GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Students will demonstrate command of the ELG by:

- Identifying the shapes of two-dimensional cross-sections of three-dimensional objects.
- Identifying three-dimensional objects generated by rotations of two-dimensional objects.

Vocabulary:

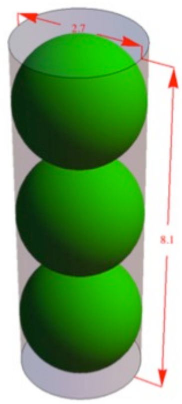
- Cross section
- Cylinder
- Pyramid
- Rotation
- Sphere

Sample Instructional/Assessment Tasks:

1) **Standard(s): G-GMD.B.4, G-GMD.A.1**
Source: https://www.illustrativemathematics.org/content-standards/HSG/GMD/B/4/tasks/512

Item Prompt:

The official diameter of a tennis ball, as defined by the International Tennis Federation, is at least 2.575 inches and at most 2.700 inches. Tennis balls are sold in cylindrical containers that contain three balls each. To model the container and the balls in it, we will assume that the balls are 2.7 inches in diameter and that the container is a cylinder the interior of which measures 2.7 inches in diameter and 3×2.7=8.1 inches high.

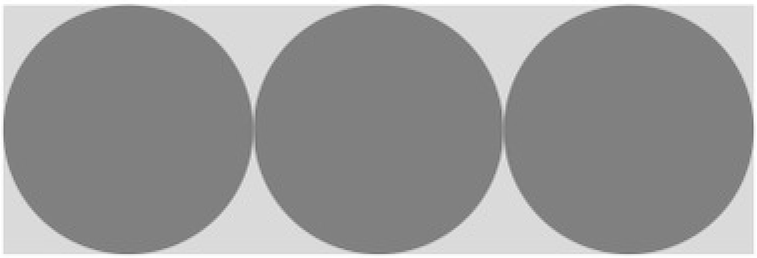


a. Lying on its side, the container passes through an X-ray scanner in an airport. If the material of the container is opaque to X-rays, what outline will appear? With what dimensions?
b. If the material of the container is partially opaque to X-rays and the material of the balls is completely opaque to X-rays, what will the outline look like (still assuming the can is lying on its side)?
c. The *central axis* of the container is a line that passes through the centers of the top and bottom. If one cuts the container and balls by a plane passing through the central axis, what does the intersection of the plane with the container and balls look like? (The intersection is also called a *cross section*. Imagine putting the cut surface on an ink pad and then stamping a piece of paper. The stamped image is a picture of the intersection.)
d. If the can is cut by a plane parallel to the central axis, but at a distance of 1 inch from the axis, what will the intersection of this plane with the container and balls look like?
e. If the can is cut by a plane parallel to one end of the can—a horizontal plane—what are the possible appearances of the intersections?
f. A cross-section by a horizontal plane at a height of $1.35+w$ inches from the bottom is made, with $0<w<1.35$ (so the bottom ball is cut). What is the area of the portion of the cross section inside the container but outside the tennis ball?
g. Suppose the can is cut by a plane parallel to the central axis but at a distance of w inches from the axis ($0<w<1.35$). What fractional part of the cross section of the container is inside of a tennis ball?

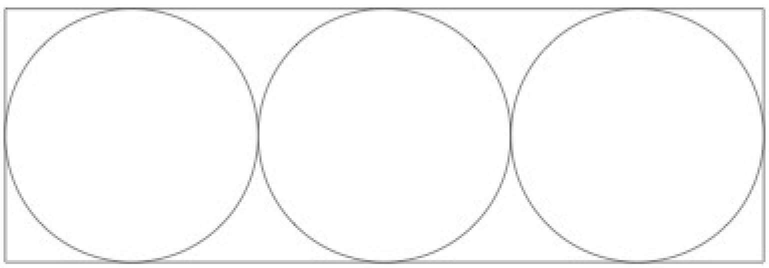
Solution:

a. The shadow is a rectangle measuring 2.7 inches by 8.1 inches.

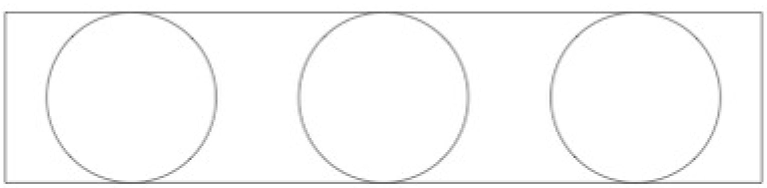
b. The shadow is a light rectangle (2.7 × 8.1 inches) with three disks inside. It looks like a traffic light:



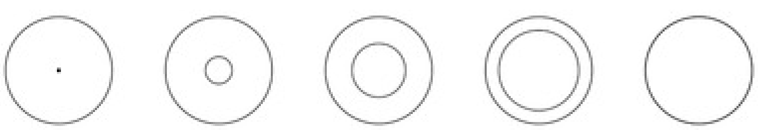
c. The image is similar to the previous one, but now only the outlines are seen:



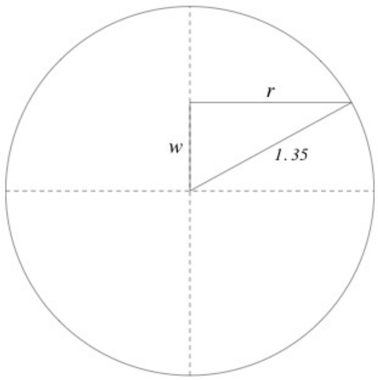
d. The intersection with the container is a narrower rectangle. The intersections with the balls are smaller circles. Because each ball touches the container along its whole "equator," the circles must touch the long sides of the rectangle:



e. The intersections are two concentric circles, except when w=0, 2.7, 5.4, 8.1 and when w=1.35, 4.05, 6.75. In the former case, we see a circle (from the container) and a point (where the plane touches a sphere). In the latter case, we see a single circle corresponding to a place where the equator of a ball touches the container.



f. The intersection of the plane with the interior of the *container* is a disk of radius 1.35 inches. Its area is $\pi(1.35)^2$ in^2. The intersection with the *ball* is a smaller disk that is contained in the first disk. The radius r of the smaller disk is the square root of $(1.35)^2-w^2$, as we see from the diagram below depicting the intersection of a plane through the central axis of the container with the bottom ball. Thus, the area of the smaller disk is $\pi((1.35)^2-w^2)$. Accordingly, the area inside the larger disk but outside the smaller is πw^2, provided that $0\le w\le 1.35$. (It is notable that the radius of the ball does not appear explicitly in the expression for this annular area.)



g. Referring to Problem d), we see that we wish to find the ratio of the total area of three congruent disks to the area of a rectangle, one of whose dimensions is equal to the diameter of the disks. The same picture used in the previous problem, but interpreted as a view from one end of the container, gives us the radius of the small disks — namely, $\sqrt{(1.35)^2 - w^2}$, so the total area of the disks is $3\pi((1.35)^2 - w^2)$. The area of the rectangle is $(8.1)2\sqrt{(1.35)^2 - w^2}$. So, the ratio is $\frac{3\pi((1.35)^2-w^2)}{(8.1)2\sqrt{(1.35)^2-w^2}} = \frac{\pi\sqrt{(1.35)^2-w^2}}{5.4}$

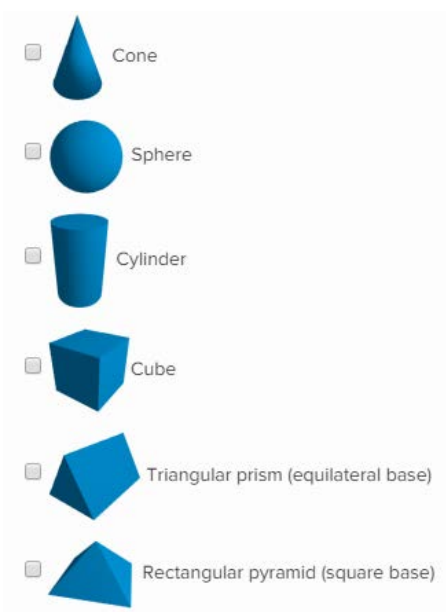
2) Standard(s) : G-GMD.B.4

Source: https://www.khanacademy.org/math/geometry/basic-geometry/cross-sections/e/cross-sections-of-3d-shapes

Item Prompt:

Which of the objects shown below could be sliced to create isosceles triangle cross-sections? Select all that apply.

*Please note that the height of the cylinder is greater than the diameter of its base, the height of the cone is greater than the diameter of its base, and the length of the prism is greater than any side of its equilateral base.

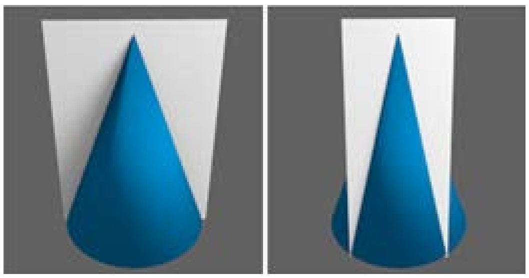


Solution:

We can create isosceles triangle cross-sections by slicing the following objects:

- Cone
- Cube
- Triangular prism
- Rectangular pyramid

To get a triangle by slicing a cone, we must slice through the exact tip of the cone. Because the distance from the tip to any point on the circular base is always the same, all triangles created by slicing cones are isosceles.



A cube can be slided to get an iscosceles triangle.

A triangular prism (with equilateral base) can be slided to get an isosceles triangle.

A rectangular pyramid (with square base) can be slided to get an isosceles triangle.

.