Vertical Progression:

7th Grade	**7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** o **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
8th Grade	**8.G.C Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.** o **8.G.C.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
Geometry	**ELG.MA.HS.G.15 Apply geometric concepts in modeling situations** o **G-MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). o **G-MG.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). o **G-MG.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Students will demonstrate command of the ELG by:

- Using geometric shapes, their measures, and their properties to describe objects.
- Applying concepts of density based on area and volume in modeling situations.
- Applying geometric methods to solve design problems.

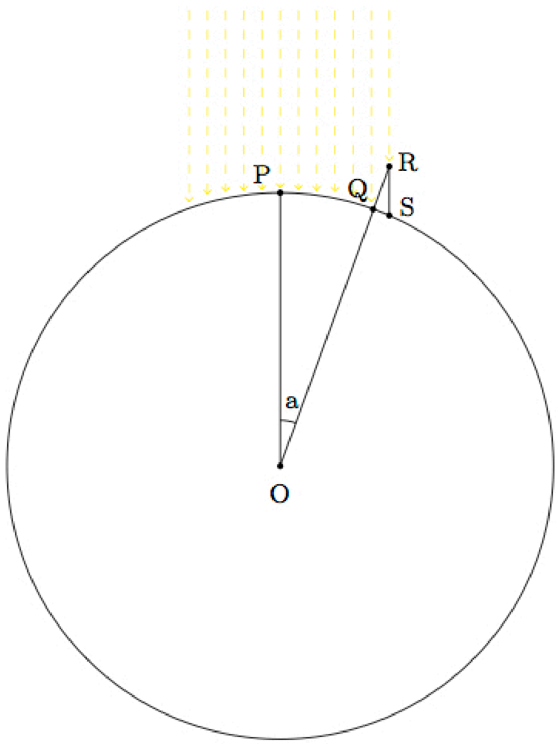
Vocabulary:

- area
- density
- model
- volume

Sample Instructional/Assessment Tasks:

1) **Standard(s): G-MG.1**
Source: https://www.illustrativemathematics.org/content-standards/HSG/MG/A/tasks/1141

Item Prompt:
The ancient Greek scientist Eratosthenes devised the following experiment for estimating the circumference of the earth, which he assumed to be spherical in shape. Pictured below is the sun's rays hitting two different locations on the surface of the earth. The point P, in the picture below, is in the tropics and, at this particular time, the sun's rays are hitting this point perpendicularly. At the point Q, the sun's rays meet the earth at an angle which can be measured by finding the length of the shadow cast by an object at point Q.



a. Assuming the rays of the sun hitting points P and R are parallel as in the picture, explain why angle POQ is congruent to angle QRS.

b. For the two locations used by Eratosthenes, the shadow cast by a ten foot pole at location Q was about 1.26 feet. Using this information, find a, the measure of angle POQ.

c. According to Eratosthenes, the distance from point P to point Q was approximately 2,584,000 feet. Using this information and the calculation from part (b) what estimate does this give for the circumference of the earth, in both feet and miles?

d. Current estimates for the earth's circumference are about 24,900 miles. Within what percent error from this current value is Eratosthenes' estimate?

Solution:

a. The rays of the sun hitting the earth are assumed to be parallel: this is a reasonable hypothesis because their point of origin, the sun, is so far away from the earth that they are very close to being parallel. Points O and P are collinear with one ray of the sun while points R and S are collinear with a second ray. So lines OP and RS are parallel. Since segment OR is a transverse of these parallel lines it follows that alternate interior angles POQ and ORS are congruent. Angle QRS is the same as angle ORS so angles POQ and ORS are congruent. This means that angle ORS has measure *a*.

b. The pole may be assumed to make a right angle with the ground so triangle RQS is a right triangle with right angle RQS and hypotenuse RS. We are given that $|RQ|=10$ feet while $|QS|=1.26$ feet. Using the Pythagorean theorem, we find $|RS|^2=|RQ|^2+|QS|^2=10^2+1.26^2$.

Using a calculator we find that |RS| is about 10.08 feet. So $\frac{|QS|}{|RS|} \approx \frac{1.26}{10.08} = 0.125$

Now $\frac{|QS|}{|RS|}$ represents the length of the side opposite angle QRS divided by the length of the hypotenuse of triangle QRS. This is the sine of the angle QRS. With a calculator we can find that angle QRS measures about 7.2 degrees. There is a subtlety here which has not been taken into account, namely that the earth is ''curved'' and so the surface of the earth going from P to Q is a small arc rather than a line segment. The circumference of the earth is so large, however, compared to the length of the shadow of the pole, that we may assume that PQ is a line segment as is done above.

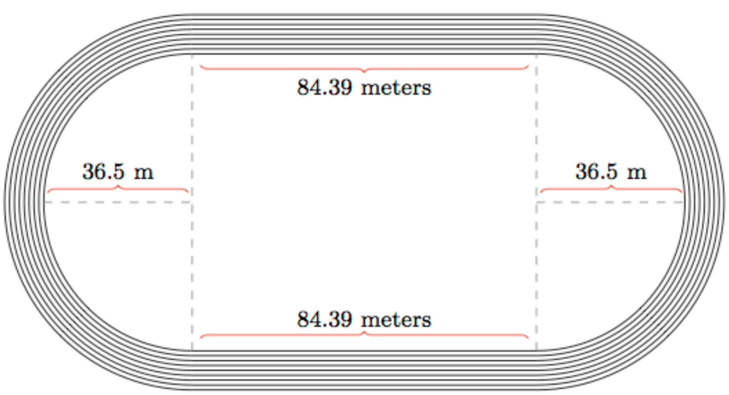
c. There are 360 degrees around the circle representing the earth and so the 7.2 degrees between P and Q represents 1/50 of the full circumference. So if the distance from P to Q is estimated at 2,584,000 feet then the full circumference of the earth would be about $50\times2,584,000$ feet=129,200,000 feet. To convert this to miles we have 129,200,000 feet=129,200,000 feet$\times1/5280$ miles/foot$\approx$24,470 miles.

d. This differs by about 430 miles from the current estimate of the circumference of the earth or about 1.7 percent.

2) **Standard(s) : G-MG.1**

Source: https://www.illustrativemathematics.org/content-standards/HSG/MG/A/tasks/1127

An Olympic 400 meter track is made up of two straight sides, each measuring 84.39 meters in length, and two semi-circular curves with a radius of 36.5 meters as pictured below:



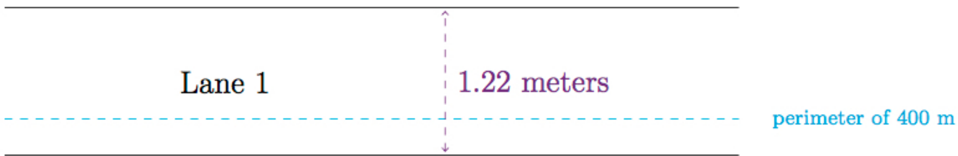
The picture is drawn to scale with one centimeter in the picture representing 20 meters on an actual Olympic track. The one of the eight lanes which is closest to the center of the track is called the first lane.

a. What is the perimeter of the track, measured on the innermost part of the first lane?

b. Each lane on the track is 1.22 meters wide. What is the perimeter of the track measured on the outermost part of the first lane?

c. In order to run the intended 400 meters in a lap, how far away from the inside of the first lane would a runner need to be?

Below is an enlarged picture of one of the straight sections of the track with the blue line representing the line around the track with perimeter exactly 400 meters:



Solution:

a. The two straightaway sections of the track are each 84.39 meters. The two semi-circular sections can be joined to form a circle whose radius is 36.5 meters and so the diameter of this circle is $2\times36.5=73$meters. The circumference of this circle will be $\pi\times73$ meters and so the total perimeter of the track is $2\times84.39+\pi\times73\approx398.12$. So the perimeter of the track is less than 400 meters.

b. For the first lane on the track, the straightaway sections are each 84.39 meters long. However, the curved sections form a circle whose radius is now $36.5+1.22=37.72$ meters. The diameter of the circle will be $2\times37.72=75.44$ meters. So the perimeter of lane 1 is $2\times84.39+\pi\times75.44\approx405.78$.

So the perimeter of lane 1 on the track is more than 400 meters and is almost 8 meters more than the perimeter of the inside of the track.

c. Suppose we let x denote the distance from the inside of lane 1 which gives a perimeter of 400 meters. This perimeter will be consist of the two straight sections which contribute 2×84.39 meters to the perimeter. In addition, there will be two semi-circular sections of radius $36.5+x$ meters. Combining these gives a circle whose diameter is $2\times(36.5+x)$ meters. So we want $2\times84.39+\pi\times2\times(36.5+x)=400$. Rewriting this we find $2\pi\times x=400-2\times84.39-2\pi\times36.5$. Solving for x we find $x\approx0.30$.

Note that this value for x is not exact but approximate. It is accurate to within about two ten thousandths of a meter or a fraction of a millimeter. So approximately 30 centimeters from the inside of lane 1 the perimeter of the track is 400 meters.