

Vertical Progression:

7 th Grade	<p>7.G.A Draw, construct, and describe geometrical figures and describe the relationships between them.</p> <ul style="list-style-type: none"> ○ 7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
8 th Grade	<p>8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.</p> <ul style="list-style-type: none"> ○ 8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations: <ul style="list-style-type: none"> ○ 8.G.A.1.a Lines are taken to lines, and line segments to line segments of the same length. ○ 8.G.A.1.b Angles are taken to angles of the same measure. ○ 8.G.A.1.c Parallel lines are taken to parallel lines. ○ 8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. ○ 8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
Geometry	<p>Congruence — G-CO</p> <p>ELG.MA.HS.G.2 Understand congruence in terms of rigid motions.</p> <ul style="list-style-type: none"> ○ G-CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. ○ G-CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. ○ G-CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Students will demonstrate command of the ELG by:

- Using geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure.
- Using the definition of congruence in terms of rigid motions to decide if two figures are congruent.
- Using the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- Explaining how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Vocabulary:

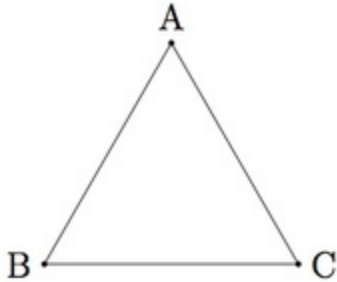
- angle-side-angle (ASA)
- congruent
- corresponding angle
- corresponding side
- rigid transformation
- side-angle-side (SAS)
- side-side-side (SSS)

Sample Instructional/Assessment Tasks:

1) Standard(s): G-CO.B

Source: <https://www.illustrativemathematics.org/content-standards/HSG/CO/B/tasks/982>

Item Prompt: Suppose ABC is an equilateral triangle:

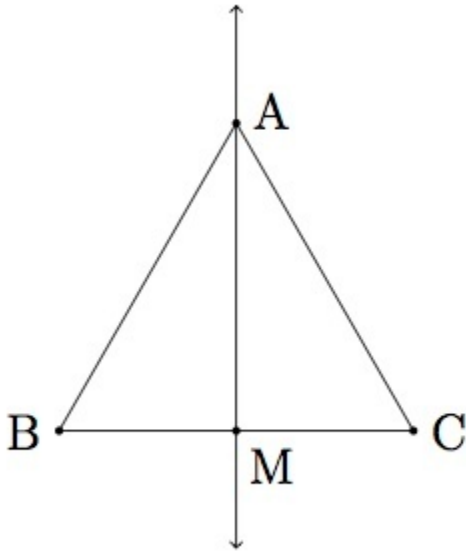


- Describe three reflections of the plane which preserve triangle ABC.
- Show that the three lines of symmetry from part (a) are the perpendicular bisectors of segments AB, AC, and BC.
- Prove that a triangle DEF is equilateral if and only if there are three reflections of the plane which preserve triangle DEF.

Solution:

- Since triangle ABC is equilateral, the segments AB, AC and BC are all congruent. Corresponding to each of the three pair of congruent sides of ABC is a reflection of the plane which preserves triangle ABC and interchanges the chosen pair congruent sides. These three lines of reflection are
 - the line through A and the midpoint of segment BC
 - the line through B and the midpoint of segment AC
 - the line through C and the midpoint of segment AB

In each case, the reflection preserves triangle ABC provided the line in question is the perpendicular bisector of the appropriate side of the triangle. For convenience, we reproduce here the full argument for case 1. Call M the midpoint of segment BC as in the picture below:



We wish to show that line AM is the perpendicular bisector of BC . It passes through the midpoint of BC so it is sufficient to show that line AM is perpendicular to BC . We will accomplish this by showing that triangles AMB and AMC are congruent. Since angles AMB and AMC make a line and are congruent, they must each be 90 degree angles. To see why triangles AMB and AMC are congruent, note that segment AB is congruent to segment AC since triangle ABC is equilateral. Segment MB is congruent to segment MC because M is the midpoint of BC . Finally, AM is congruent to AM and so by SSS triangles AMB and AMC are congruent.

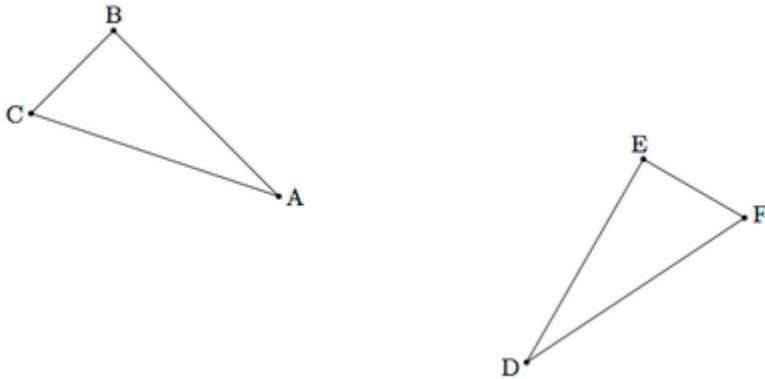
- b.** The lines from part (a) are the perpendicular bisectors of the three sides AB , AC , and BC because of the definition of reflections. If a reflection preserves a triangle, it must pass through one vertex and exchange the other two. The reflection which accomplishes this is the one about the perpendicular bisector of the appropriate side of the triangle.
- c.** The first two parts of this task show that when DEF is an equilateral triangle, there are three reflections of the plane which preserve DEF . The only possible reflections of the plane which preserve triangle DEF are the reflections about the perpendicular bisectors of the three sides of the triangle. If all three of these reflections preserve triangle DEF this means that all three sides are congruent: given two sides of the triangle, DE and DF for example, the reflection about the line joining D and the midpoint of EF will map side DE to side DF and vice versa making these two sides congruent. Arguing in the same way for sides DF and EF will show that they are also congruent and so triangle DEF is equilateral.

2) Standard(s): G-CO-B

Source: <https://www.illustrativemathematics.org/content-standards/HSG/CO/B/8/tasks/110>

Item Prompt:

In the picture below segment AB is congruent to segment DE, segment AC is congruent to segment DF and segment BC is congruent to segment EF:

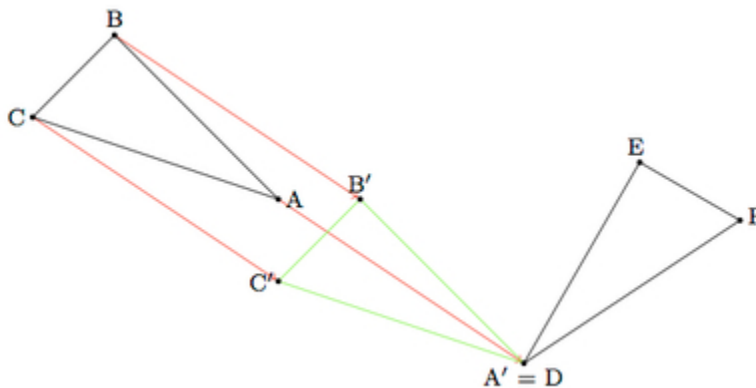


Show that the two triangles ABC and DEF are congruent via the following steps, which produce a rigid transformation of the plane sending $\triangle ABC$ to $\triangle DEF$.

- Show that there is a translation of the plane which maps A to D. Call B' and C' the images of B and C under this transformation.
- Show that there is a rotation of the plane which does not move D and which maps B' to E. Call C'' the image of C' under this transformation.
- Show that there is a reflection of the plane which does not move D or E and which maps C'' to F.

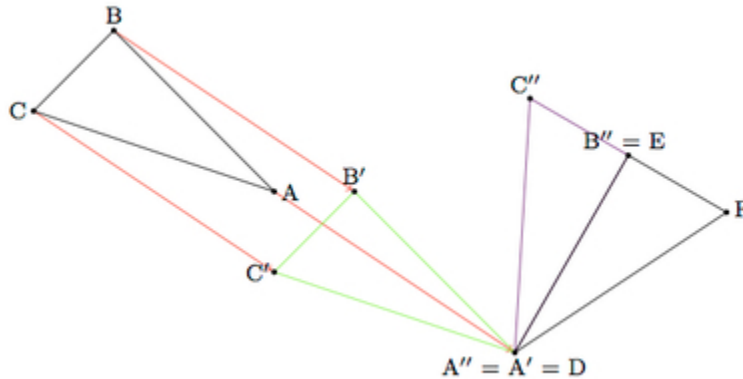
Solution:

- Translation by the segment AD maps A to D. The displacement is shown by the red arrows in the picture below:



The result of the translation is $\triangle A'B'C'$ shown in green.

- b. In order to move B' to E , without moving D , we rotate triangle $A'B'C'$ about point D by angle $B'A'E$, the effect of which is pictured below (the point C'' is the image of C' under this rotation):



- We know that the image of B' , under rotation about D by angle $B'A'E$, will be E because, by assumption, segment AB is congruent to segment DE .
- c. Now it remains to move C'' to F , without moving D or E . The only possible rigid motion of the plane other than the identity which does not move D or E , is reflection about segment DE so we must check that this reflection moves C'' to F . We know that $|DF| = |A''C''|$ because translations and rotations preserve distances and $|AC| = |DF|$. Similar reasoning shows that $|EF| = |B''C''|$. Hence $\triangle DFC''$ is isosceles with $m(\angle DFC'') = m(\angle DC''F)$. Similarly $m(\angle EFC'') = m(\angle EC''F)$. This means that $m(\angle F) = m(\angle C'')$. By SAS $\triangle DEF$ is congruent to $\triangle DB''C''$. This means that segment DE is the perpendicular bisector of segment $C''F$. Reflection over segment $C''F$ maps C'' to F and shows that $\triangle ABC$ is congruent to $\triangle DEF$.