Geometry

ELG HS.G.4: Make geometric constructions.

Vertical Progression:

7th Grade	**7.G.A Draw, construct, and describe geometrical figures and describe the relationships between them.** o **7.G.A.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
8th Grade	**8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.** o **8.G.A.1** Verify experimentally the properties of rotations, reflections, and translations: o **8.G.A.1a** Lines are taken to lines, and line segments to line segments of the same length. o **8.G.A.1b** Angles are taken to angles of the same measure. o **8.G.A.1c** Parallel lines are taken to parallel lines. o **8.G.A.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
Geometry	**ELG.MA.HS.G.4 Make geometric constructions** o **G-CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* o **G-CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Students will demonstrate command of the ELG by:

- Making formal geometric constructions with a variety of tools and methods.
- Constructing an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
- Copying a segment and copying an angle.
- Bisecting a segment and bisecting an angle.
- Constructing perpendicular lines, including the perpendicular bisector of a line segment.
- Constructing a line parallel to a given line through a point not on the line.

Vocabulary:

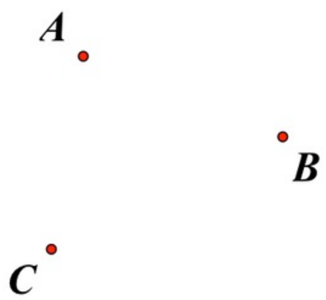
- angle bisector
- compass
- geometric Construction
- inscribed
- parallel line
- perpendicular bisector
- regular hexagon
- straightedge

Sample Instructional/Assessment Tasks:

1) **Standard(s): G-CO.D, G-C.A.3**
Source: https://www.illustrativemathematics.org/content-standards/HSG/CO/D/tasks/508

Item Prompt:

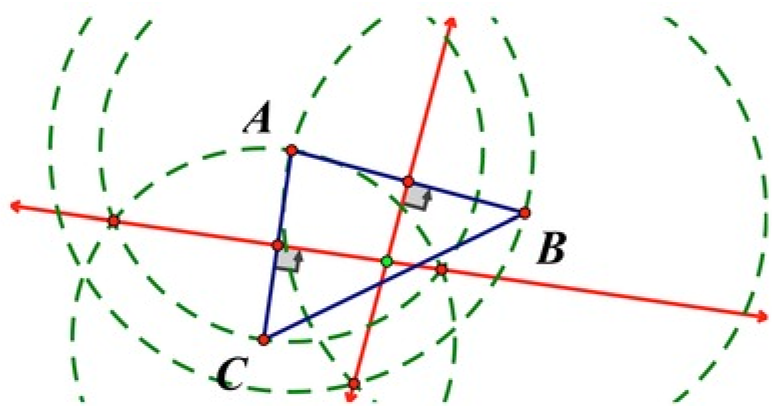
You have been asked to place a fire hydrant so that it is an equal distance form three locations indicated on the following map.



a. Show how to fold your paper to physically construct this point as an intersection of two creases.
b. Explain why the above construction works, and in particular why you only needed to make two creases.

Solution:

a. Fold and crease the paper so that line segment point A lands onto point B. Do the same so that point A lands on point C. The intersection of the two creases is the point we want.
b. Since the desired location is an equal distance from three non-collinear points, we are looking for the center of the circle passing through these three points. This corresponds to the center of the circle circumscribed about the triangle ABC. The center of the circumscribed circle, called the circumcenter, can be found by constructing the perpendicular bisectors of the three sides of the triangle (precisely the creases made in the paper on the previous step). Since the perpendicular bisectors are concurrent, it is sufficient to construct only two of the three perpendicular bisectors.



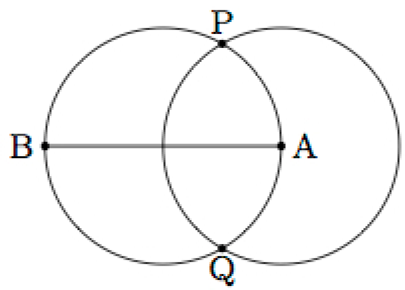
The concurrency of the perpendicular bisectors can be argued as follows: Let P be the green dot in the above diagram, the intersection of the perpendicular bisectors of AB and AC. By virtue of P being on the perpendicular bisector of AB, P is equidistant from A and B, i.e., PA=PB. Similarly, by virtue of being on the perpendicular bisector of AC, we have PA=PC. But this implies that PB=PC, i.e., that P is also on the perpendicular bisector of BC, demonstrating that P indeed lies on all three perpendicular bisectors.

2) Standard(s): G-CO.D.13

Source: https://www.illustrativemathematics.org/content-standards/HSG/CO/D/13/tasks/1557

Item Prompt:

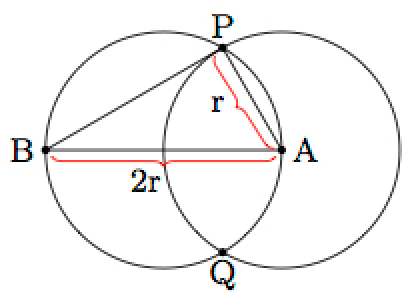
Suppose we are given a circle of radius r. The goal of this task is to construct an equilateral triangle whose three vertices lie on the circle. Suppose segment AB is a diameter of the circle. Draw a circle with center A and radius r and label the two points of intersection of the circles P and Q as pictured below:



a. Show that m∠ABP=30 degrees and m∠ABQ=30 degrees.
b. Show that △PBQ is an equilateral triangle inscribed in the given circle.

Solution:

a. We know that |AP|=r because it is a radius of our auxiliary circle. We also know that |AB|=2r since it a diameter of our given circle of radius r:



Thus

|AP||AB|=12.

Since △ABP is inscribed in a circle and side AB is a diameter of this circle, this means that ∠APB is a right angle. Side AP is the side opposite angle ABP. Therefore, sin∠ABP=|AP||AB|=12. This means that ∠ABP is a 30 degree angle.

We can apply the same argument to show that ∠*ABQ* is a 30 degree angle. Alternatively, since *AB* contains the centers of both circles, reflection about AB maps the two circles to themselves and interchanges P and Q. This means that reflection about AB maps ∠*ABP* to ∠*ABQ*. Since reflections preserve angle measurements, this means that m∠*ABQ*=m∠*ABP*=30 degrees.

b. We know that m∠*PBQ*=m∠*ABQ*+m∠*ABP*. We have found that the angles *ABQ* and *ABP* each measure 30 degrees so m∠*PBQ*=60 degrees. We also know that △*PBQ* is isosceles with *PB*=QB. This is true because reflection about AB maps PB to QB and vice versa. Base angles in an isosceles triangle are congruent so m∠BPQ=m∠BQP) We also know that m∠PBQ+m∠BPQ+m∠BQP=180 degrees. Using the fact that m∠PBQ=60 degrees, we find that m∠BPQ+m∠BQP=120 degrees.

Since these two angles are congruent we find that m∠*BPQ*=m∠*BQP*=60 degrees. Since all three angles in △*PBQ* are 60 degree angles it is an equilateral triangle. The vertices *P,B,Q* all lie on the circle so △*PBQ* is an equilateral triangle inscribed in the given circle. It is pictured below with all of the auxiliary constructions removed: