Geometry

ELG HS.G.5: **Understand similarity in terms of similarity transformations.**

Vertical Progression:

7th Grade	**7.RP.A Analyze proportional relationships and use them to solve real-world and mathematical problems.** o **7.RP.A.2** Recognize and represent proportional relationships between quantities. o **7.RP.A.2d** Explain what a point (*x, y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate. **7.G.A Draw, construct, and describe geometrical figures and describe the relationships between them.** o **7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
8th Grade	**8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.** o **8.G.A.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. o **8.G.A.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
Geometry	**ELG.MA.HS.G.5 Understand similarity in terms of similarity transformations.** o **G-SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor: o **G-SRT.1a** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. o **G-SRT.1b** The dilation of a line segment is longer or shorter in the ratio given by the scale factor. o **G-SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. o **G-SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Students will demonstrate command of the ELG by:

- Verifying the properties of dilation for lines passing through the center of dilation and not passing through the center.
- Using transformations to determine/prove that two figures are similar or not.
- Use transformations to prove that two triangles with two pairs of congruent angles are similar.

Vocabulary:

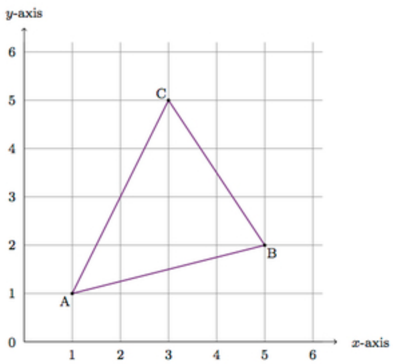
- dilation
- proportional
- scale factor
- similar
- transformation

Sample Instructional/Assessment Tasks:

1) **Standard(s): G-SRT.A**
Source: https://www.illustrativemathematics.org/content-standards/HSG/GPE/B/6/tasks/1867

Item Prompt:

Below is a picture of △ABC with vertices lying on grid points:



a. Draw the image of $\triangle ABC$ when it is scaled with a scale factor of $\frac{1}{3}$ about the vertex A. Label this triangle $A'B'C'$ and find the coordinates of the points A', B', and C'.

b. Draw the image of $\triangle ABC$ when it is scaled with a scale factor of $\frac{2}{3}$ about the vertex B. Label this triangle $A''B''C''$ and find the coordinates of the points A'', B'', and C''.

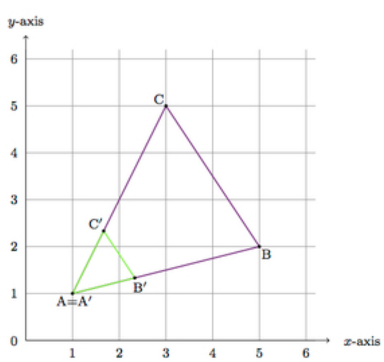
c. How does A'' compare B'? Why?

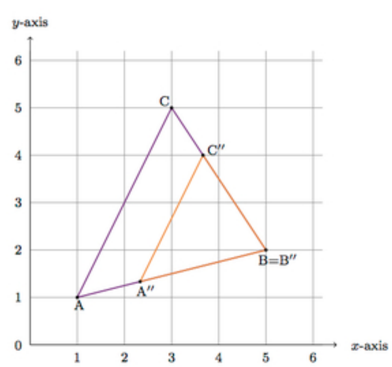
Correct Answer:

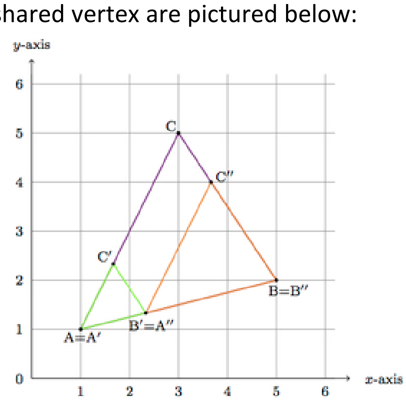
a. Since A is the center of the dilation, it does not move and we will have A′=A. For vertex B, we know that B′ will lie on $\overleftrightarrow{AB}$ because dilations preserve lines through the center of dilation. Since the scale factor is $\frac{1}{3}$ we know that $|AB'| = \frac{1}{3}|AB|$. From A to B is 4 units to the right and one unit up. Therefore, one third of the way from A to B (along $\overline{AB}$) will be the point $B' = \left(1 + \frac{4}{3}, 1 + \frac{1}{3}\right)$. Applying this technique to C which is 2 units to the right and 4 units up from A, we find $C' = \left(1\frac{2}{3}, 2\frac{1}{3}\right)$. The scaled triangle A′B′C′ is pictured below:

b. Since B is the center of the dilation, it does not move and we will have B′′=B. For vertex A, we know that A′′ will lie on $\overleftrightarrow{AB}$ because dilations preserve lines through the center of dilation. Since the scale factor is $\frac{2}{3}$ we know that $|BA''| = \frac{2}{3}|BA|$. From B to A is 4 units to the left and one unit down. Therefore, two thirds of the way from B to A (along $\overline{AB}$) will be the point $A'' = \left(4 - \frac{8}{3}, 2 - \frac{2}{3}\right)$. Applying this technique to C which is 2 units to the left and 3 units up from B, we find $C'' = \left(3\frac{2}{3}, 4\right)$. The scaled triangle A′′B′′C′′ is pictured below:

c. The calculations above show that $B' = A'' = \left(2\frac{1}{3}, 1\frac{1}{3}\right)$. To see why these vertices are the same, note that the sum of the dilation factors $\frac{1}{3}$ and $\frac{2}{3}$ is 1. The dilation about A maps $\overline{AB}$ to the first third of $\overline{AB}$ while the dilation about B maps $\overline{AB}$ to the second two thirds of this segment. The two scaled triangles with a shared vertex are pictured below:



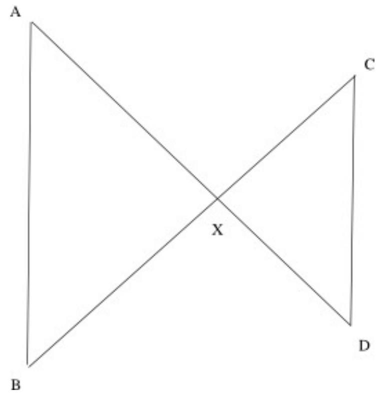




2) Standard(s): G-SRT.2

Source: https://www.illustrativemathematics.org/content-standards/HSG/SRT/A/2/tasks/603

Item Prompt:
In the picture given below, line segments AD and BC intersect at X. Line segments AB and CD are drawn, forming two triangles AXB and CXD.



In each part (a)-(d) below, some additional assumptions about the picture are given. In each problem, determine whether the given assumptions are enough to prove that the two triangles are similar; and if so, what the correct correspondence of vertices is. If the two triangles must be similar, prove this result by describing a sequence of similarity transformations that maps one triangle to the other. If not, explain why not.

a. The lengths AX and XD satisfy the equation $2AX=3XD$.
b. The lengths AX, BX, CX, and DX satisfy the equation $AX/BX=DX/CX$.
c. Lines *AB* and *CD* are parallel.
d. Angle *XAB* is congruent to angle *XCD*.

Solution:

a. We are given that 2AX = 3XD. This is not enough information to prove similarity. To see that in a simple way draw an arbitrary triangle $\triangle$AXB. Extend AX and choose a point D on the extended line so that 2AX=3XD. Extend BX and choose a point C on the extended line so that 2BX=XC. Now triangles AXB and CXD satisfy the given conditions but are not similar. (If you are extremely unlucky, AXB and CXD might be similar by a different correspondence of sides. If this happens, rotate the line BC a little bit. The lengths of AX, XD, BX, XC remain the same but the triangles are no longer similar.)

b. We are given that $\frac{AX}{BX} = \frac{DX}{CX}$. Rearranging this proportion gives $\frac{AX}{DX} = \frac{BX}{CX}$. Let $k = \frac{AX}{DX}$. Suppose we rotate the triangle DXC 180 degrees about point X: Since AD is a straight line, DX and AX align upon rotation of 180 degrees, as do CX and BX, and so angles DXC and AXB coincide after this rotation. (Alternatively, one could observe that DXC and AXB are vertical angles, and hence congruent, giving a second argument the angles line up precisely). Then dilate the triangle DXC by a factor of k about the center X. This dilation moves the point D to A, since k(DX) = AX, and moves C to B, since k(CX) = BX. Then since the dilation fixes X, and dilations take line segments to line segments, we see that the triangle DXC is mapped to triangle AXB. So the original triangle DXC is similar to AXB. (Note that we state the similarity so that the vertices of each triangle are written in corresponding order.)

c. Again, rotate triangle DXC so that angle DXC coincides with angle AXB. Then the image of the side CD under this rotation is parallel to the original side CD, so the new side CD is still parallel to side AB. Now, apply a dilation about point X that moves the vertex C to point B. This dilation moves the line CD to a line through B parallel to the previous line CD. We already know that line AB is parallel to CD, so the dilation must move the line CD onto the line AB. Since the dilation moves D to a point on the ray XA and on the line AB, D must move to A. Therefore, the rotation and dilation map the triangle DXC to the triangle AXB. Thus DXC is similar to AXB.

d. Suppose we draw the bisector of angle AXC, and reflect the triangle CXD across this angle bisector. This maps the segment XC onto the segment XA; and since reflections preserve angles, it also maps segment XD onto segment XB. Since angle XCD is congruent to angle XAB, we also know that the image of side CD is parallel to side AB. Therefore, if we apply a dilation about the point X that takes the new point C to A, then the new line CD will be mapped onto the line AB, by the same reasoning used in (c). Therefore, the new point D is mapped to B, and thus the triangle XCD is mapped to triangle XAB. So triangle XCD is similar to triangle XAB. (Note that this is not the same correspondence we had in parts (b) and (c)!)