ELG HS.G.6: **Prove theorems involving similarity.**

Vertical Progression:

7th Grade	**7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** o **7.G.B.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
8th Grade	**8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.** o **8.G.A.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. **8.G.B Understand and apply the Pythagorean Theorem.** o **8.G.B.6** Explain a proof of the Pythagorean Theorem and its converse.
Geometry	**ELG.MA.HS.G.6 Prove theorems involving similarity.** o **G-SRT.4** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity*. o **G-SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Algebra 2	**ELG.MA.HS.F.10 Prove and apply trigonometric identities.** o **F-TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
	ELG.MA.HS.G.8 Apply trigonometry to general triangles. o **G-SRT.10 (+)**Prove the Laws of Sines and Cosines and use them to solve problems.

Students will demonstrate command of the ELG by:

- Proving theorems about triangles involving similarity (paragraph, flow-chart, or two-column proof).
- Using congruence and similarity criteria for triangles to solve problems.
- Using congruence and similarity criteria for triangles to prove geometric figure relationships.

Vocabulary:

- converse
- parallel
- proportional
- Pythagorean theorem
- right triangle
- similarity

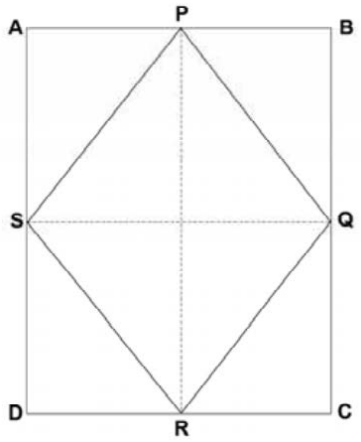
Sample Instructional/Assessment Tasks:

1) **Standard(s): G-SRT.2, G-SRT.5**

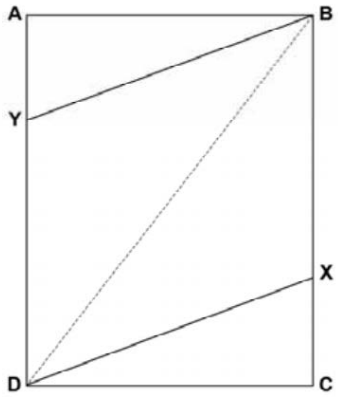
Source: http://www.insidemathematics.org/assets/common-core-math-tasks/rhombuses.pdf

Item Prompt:

1. **Quadrilateral PQRS is drawn in an 8cm by 10cm rectangle ABCD. P, Q, R and S are the midpoints of the sides of the rectangle. What specific quadrilateral is PQRS? Explain why.**



2. Here is parallelogram BXDY drawn in the 8cm by 10cm rectangle ABCD. Show that BXDY is a rhombus if AY and CX are 1.8 cm.



3. Show that PQRS and BXDY are similar. Explain your reasoning.

Correct Answer:

1. **Quadrilateral PQRS is a rhombus because it has four congruent sides.**
2. **BY = XD = 8.2 cm (steps for work are shown below)**
   a. **$BY^2 = AY^2 + AB^2$**
   b. **$BY^2 = 1.8^2 + 8^2$**
   c. **BY = 8.2**

3.

In rhombus PQRS, shows that PS = 6.4
In rhombus BXDY, shows that BD = 12.8

Shows that the sides of the triangles PSR and BYD are proportional.
Shows that:
$\frac{PS}{BY} = \frac{PR}{BD}$
e.g. accept
$\frac{6.4}{8.2} = 0.78$
and
$\frac{10}{12.8} = 0.78$
Since the sides are proportional, the rhombuses are similar.

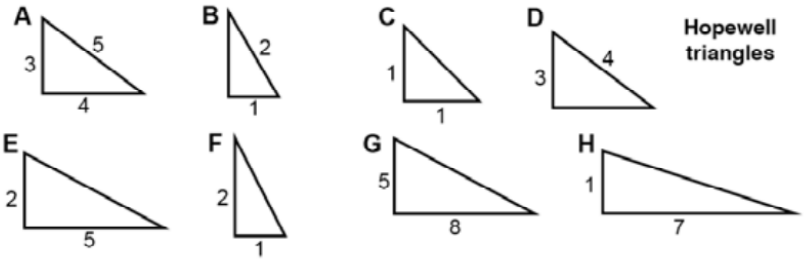
Alternatively
Finds angle SPQ = angle YBX = 77.3°
Or angle PSR = angle BYD = 102.7°
Since the angles are equal, the rhombuses are similar.

2) Standard(s): G-SRT.2, G-SRT.5, G-SRT.6, G-SRT.7, G-SRT.8

Source: http://www.insidemathematics.org/assets/common-core-math-tasks/hopewell%20geometry.pdf

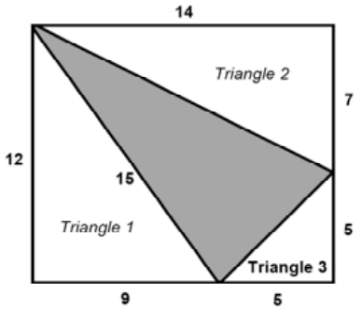
Item Prompt:

The Hopewell people were Native Americans whose culture flourished in the central Ohio Valley about 2000 years ago. The Hopewell people constructed earthworks using right triangles, including those below.



1. What is the length of the hypotenuse of Triangle H? Give your answer correct to one decimal place. Show your calculation.
2. What is the size of the smallest angle in Triangle A? Give your answer correct to one decimal place. Show your calculation.

The diagram below shows the layout of some Hopewell earthworks. The centers of the Newark Octagon, the Newark Square and the Great Circle were at the corners of the shaded triangle.



Not drawn to scale

The three right triangles surrounding the shaded triangle form a rectangle measuring 12 units by 14 units. Each of these three right triangles is similar to one of the Hopewell triangles on the previous page. For example, Triangle 3 above is similar to Hopewell Triangle C.

3. Which Hopewell triangle is similar to Triangle 1? Explain.
4. Is the shaded triangle a right triangle? Explain and show all your work.

Solution:

1. 7.1
   a. Work shown: $\sqrt{(1^2 + 7^2)}$
2. 36.8° to 36.9°
   a. Work shown: $\sin^{-1}\frac{3}{5}$ or $\cos^{-1}\frac{4}{5}$ or $\tan^{-1}\frac{3}{4}$
3. Triangle A
   a. Triangle 1 is an enlargement of Triangle A by a scale factor of 3.
4. No, because…
   a. Side lengths are $\sqrt{225}, \sqrt{50}, \sqrt{245}$ and they don't satisfy the Pythagorean Rule: 245 ≠ 225 + 50.
   b. OR used trigonometry to find angle measurements (71.6°, 81.9°, 25.5°)

   OR triangle 3 is isosceles since it has two 45° angles, and triangles 1 and 2 are not isosceles and therefore do not have 45° angles. Angle in shaded triangle = 180° - 45° - non 45° ≠ 90°